AUT Journal of Civil Engineering

AUT J. Civil Eng., 1(2) (2017) 195-204
DOI: 10.22060/ceej.2017.13005.5309

Support Vector Machine to predict the discharge coefficient of Sharp crested w-planform weirs

A. Parsaie, A. H. Haghiabi*

Water Engineering Department, Lorestan University, Khorramabad, Iran

ABSTRACT: In this paper, the discharge coefficient (C_d) of triangular labyrinth weir was predicted using Multilayer Perceptron Neural Network (MLPNN), Radial Basis Neural Network (RBFNN) and support vector machine (SVM). To this end, 223 data sets related to the effective parameters on C_d were collected. Using dimensional analysis techniques, the involved dimensionless parameters on C_d were derived. To find out the most effective parameters on C_d, the Gamma test (GT) was analyzed. Results of GT demonstrated that H/P, L_w/L_c, and L_w/W_m are the most effective parameters on C_d. To develop ANN and SVM, different types of transfer and kernel functions were tested. During the testing of transfer and kernel functions for developing the ANN and SVM models, respectively, it was found that tensing and RBFNN have the best performance for predicting the C_d. Overall evaluation of the results of developed models indicated that both models have a suitable accuracy in predicting the C_d; however, the SVM is a bit more accurate. Comparing the outcomes of the applied models in terms of DDR index shows that the data dispersivity of SVM is less than the others; therefore, this model is more reliable.

Review History:

Received: 11 June 2017
Revised: 27 August 2017
Accepted: 5 November 2017
Available Online: 25 November 2017

Keywords:

W plan form weirs
nonlinear crest
flow measurement
discharge capacity
Gamma test

1- Introduction

Labyrinth weir is a novel approach for improving the hydraulic efficiency of weirs considered instead of conventional linear weirs. Due to the effect of climate change on regime of river flow and increasing the probability of occurrence of the probable maximum flood (PMF), improving the discharge capacity of weirs specifically in earthen dam is necessary [1]. This approach has been proposed to insert the existing weirs in dam projects to improve their discharge capacity. Several approaches related to increasing the discharge capacity of weirs have been proposed whereas the labyrinth weir is the most practical approach [2]. Among non-linear weirs, labyrinth weirs have been widely welcome by researchers due to their high efficiency in term of passing the flow especially in low head projects. Comparison of labyrinth weir with conventional sharp linear weir has showed that they are more efficient about 3 to 5 times [3]. Study of labyrinth weirs has been started by Taylor [4]. He investigated several shapes such as triangular, trapezoidal and rectangular for the crest of labyrinth weirs and found that the trapezoidal crest is more efficient compared to others. From Taylor [4] to now several studies have been conducted on the labyrinth weir. In this regard, conducted studies by Hay and Taylor [2] on labyrinth weirs and their approach to design labyrinth weir can be mentioned. In follow Houston [5] assessed the method of Hay and Taylor [2] for practical purposes and found that the proposed approach of Hay and Taylor [2] include obvious errors compared to the measured data. Recently, Ghodsian [6] has investigated the effect of rounding the top of the side walls of weirs on its hydraulic efficiency. He stated that rounding the top of side wall has a significant effect on increasing its efficiency. Another type of labyrinth weirs named piano key weir has been proposed to be constructed in places that the footprint of weirs is limited such as top of the dam and narrow channels [7-11]. Due to the high performance of labyrinth weirs, these important roles in hydraulic systems and also complex hydraulic behavior of them, from 2003 to the present day, two international technical conferences have been held [12,13]. It is notable that the concept of labyrinth weir has been used for improving the discharge capacity of side weirs, as well [14]. Due to the high cost of experiments and constructing the scaled laboratory model, investigators have attempted to use mathematical methods for modeling of hydraulic properties of labyrinth weirs [15,16]. In the field of mathematical modeling using the computational fluid dynamic (CFD) and soft computing techniques as the two main parts of mathematical modeling has been reported. In the field of CFD, Robertson [17] has used Flow 3D for numerical modeling of flow over the weirs and showed that this model has suitable performance for modeling the hydraulic properties of labyrinth weirs. Using the optimized radial basis neural network for predicting the discharge coefficient of labyrinth weir has been proposed by Zaji et al. (2015). Based on those studies, the optimized radial basis function neural network has suitable performance for modeling the discharge coefficient. Reviewing the literature showed that the labyrinth weir is an accepted approach for improving the discharge capacity of existing weirs. The main

Corresponding author, E-mail: haghiabi.a@lu.ac.ir

parameter related to labyrinth weir is discharge coefficient. Hence, it is necessary to use powerful soft computing techniques such as support vector machine (SVM) methods to predict this parameter. Therefore, in this study a SVM model is prepared to predict the C_d and in the follow its performance is compared with MLPNN. The performance of developed models (MLPNN and SVM) in this study is compared with applied models in the previous studies (RBFNN model), as well. The MLPNN and SVM are powerful soft computing methods which have been widely used for function fitting, pattern recognition, image processing, etc. [15-17]. These methods have successfully applied in most areas of water engineering such as river sediment load prediction, water quality modeling, river engineering, etc. [15,18].

2- Method and Materials

Discharge coefficient of a triangular labyrinth weir is proportional to the properties of weir geometry and hydraulic conditions. The main geometrical and hydraulic parameters effective the discharge coefficient of sharp crested w-planform weirs are shown in Figure 1.

Where P is the weir height, W_{mc} is the main channel width, W_c is the width of one cycle and H is the total Head of flow. Formulation of the involved parameters on discharge coefficient is presented in Equation 1 (Carollo et al., 2012).

$$C_d = f\left(H, W_{mc}, W_c, L_w, L_c, P, g, V, \sigma, \mu, \rho\right) \tag{1}$$

In which L_w: total length of the weir, L_c: length of one cycle, V: flow velocity, g: gravitational acceleration, σ is Surface tension and ρ is the Density of flow. Using the dimensional analysis, including π theorem, the influenced dimensionless parameters on the discharge coefficient are derived as Equation 2. It is notable the flow in the channel is turbulence and usually, investigators in hydraulic experience have tried to remove the effect of surface tension, therefore, the Reynolds and Weber numbers can be negligible [6].

$$C_d = f\left(\frac{H}{P}, \frac{L_w}{W_{mc}}, \frac{L_c}{W_c}, \frac{L_w}{L_c}, \frac{W_{mc}}{W_c}\right) \tag{2}$$

Developing the soft computing techniques is based on the dataset. This means to predict a phenomenon by soft computing techniques, its behavior and influential parameters on it should be recorded previously. To predict the C_d, 223 data sets were collected from [6]; Kumar et al. [18]. The range of the collected data is given in Table 1.

Table 1. Range of collected data related to the triangular labyrinth weir

Parameters range	Min	Max	Avg	STDEV
Weir length	0.245	1.200	0.475	0.282
Channel width	0.245	0.300	0.271	0.019
Cycle width	0.123	0.280	0.213	0.075
Cycle length	0.123	1.082	0.373	0.263
Weir height	0.092	0.170	0.110	0.024
Total head	0.007	0.145	0.046	0.024
Discharge coefficient	0.148	0.906	0.595	0.172

2- 1- Gamma Test

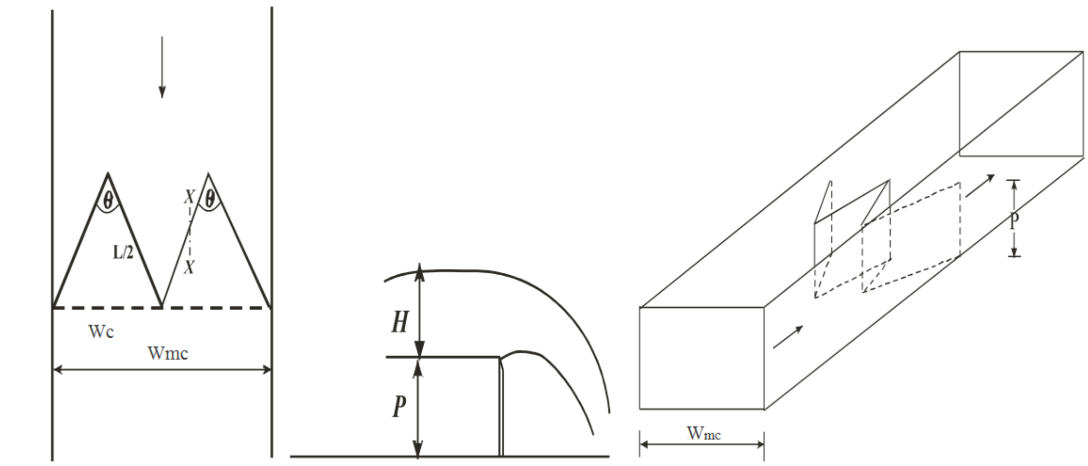
The Gamma test was used to examine the relationship between inputs and outputs in numerical data-sets without a need to construct the prediction model. The Gamma test is used to estimate the variance of the output before modeling, even though the model is unknown. This error variance estimate presents a target Mean Squared Error that any smooth non-linear function should attain on unseen data. Suppose we have a set of observed data represented by:

$$\left((x_1,\ldots,x_M), y\right) = (x, y) \tag{3}$$

where the vector $X=(x_1,\ldots,x_M)$ is the input, confined to a closed bounded set $C \in R^M$ and the scalar y is the corresponding output, without loss of generality. The only assumption made is that the relationship of the system is in the following form:

$$y = f\left(x_1,\ldots,x_M\right) + r \tag{4}$$

where f represents a smooth function and r denotes an indeterminable part, which may be due to the real noise of lack of functional determination in the assumed input/output



(a) Plan (b) Section X-X (c) 3D View of the weir and the flume

Fig. 1. Sketch of triangular labyrinth weir

relationship. The Gamma test was used to return a data-derived estimate for *Var*(*r*) without knowing the underlying function *f*, just directly from the data. The estimate of the model's output variance called the Gamma statistic and represented by *Γ* cannot be accounted for by a smooth data model. The Gamma test is derived from the Delta function of the input vectors:

$$\gamma_M(k)=\frac{1}{2M}\sum_{i=1}^{M}(y_{N[i,k]}-y_i)^2 \qquad (5)$$

where $x_{N[i,k]}$ denotes the index of the k[th] nearest neighbour to x_i, and |.| denotes Euclidean distance. Thus $\delta_M(k)$ is the mean square distance to the k[th] nearest neighbour. The corresponding Gamma function of the output values is:

$$\gamma_M(k)=\frac{1}{2M}\sum_{i=1}^{M}(y_{N[i,k]}-y_i)^2 \qquad (6)$$

The Gamma test computes the mean-squared k[th] nearest neighbour distances $\delta(k)$, $(1\le k\le k_{Max})$ and the corresponding $\gamma(p)^2$. In order to compute *Γ* the best line is constructed for the *p* points($\delta_M(k)$, $\gamma_M(k)$), and the vertical intercept, *Γ* is returned as the gamma value. The regression line slope is also returned to show the complexity of the model *f*. The V_{ratio} is the standardized results by considering *Γ/Var*(*y*). It returns a scale invariant noise estimate which normally lies between zero and one (Noori et al. 2011).

2- 2- Artificial Neural Network techniques

ANN is a non-linear mathematical model that is able to simulate arbitrarily complex non-linear processes that relate the inputs and outputs of any system. In many complex mathematical problems that lead to solving complex non-linear equations, Multilayer Perceptron Networks are the common types of ANN that are widely used in the research studies. To use MLP model, the definition of appropriate functions, weights and bias should be considered. Due to the nature of the problem, different activity functions in neurons can be used. An ANN may have one or more hidden layers. Inputs introduced to each neuron are multiplied in weights (w_i) and then summed by a constant value called bias (*b*), then passed through a transfer function. Weight and biases' values will be justified progressively and corrected during training process comparing the predicted outputs with the known outputs. Such networks are often trained using back propagation algorithm. In the present study, ANN is trained by Levenberg–Marquardt technique because this technique is more powerful and faster than the conventional gradient descent technique (Parsaie et al. 2017; Parsaie and Haghiabi 2017).

2- 3- Radial Basis Function (RBFNN) Neural Network

RBFNN is a type of MLPNN that contains only three layers with a feed-forward structure. The first layer is used for input introduction. The last layer is used for summarization of mathematical operation of the hidden layer. The hidden layer is used for main mathematical computation to map input features to the output. The hidden layer can get numbers of neurons. The governing function of neurons is Radial Basis Function (RBF). To design RBFNN, it is required to justify the number of neurons in the hidden layer. The aim of RBFNN model training is mapping the input space to output space as $f: R^n \rightarrow R$. The transfer function of the RBFNN model is defined as Equation 7.

$$f(v)=\sum_{i=1}^{n}w_i\varphi\left(\left\|v-c_i\right\|\right) \qquad (7)$$

Where *v* is the inputs variable, w_i is the weight coefficients, φ is Gaussian function, which is the basic function used as kernel function in RBFNN model development and is defined as Equation 8.

$$\varphi(v)=e^{\left(\frac{-v^2}{2\sigma^2}\right)} \qquad (8)$$

RBFNN model training usually is carried out by Gradient Descent approach. The aim of RBFNN model is defining the value of kernel function parameters and weights. The initial value of weights is defined randomly. The error for each sample of the data set is calculated by Equation 9.

$$e_i=t_i-y_i=t_i-\sum_{j=}^{N}w_j\varphi\left(\left\|v_i-c_i\right\|\right) \qquad (9)$$

The error for total input data set is calculated as Equation 10.

$$E=\frac{1}{2}\sum_{i=1}^{p}\left|e_i\right|^2 \qquad (10)$$

RBFNN model preparation is finished when the error of RBFNN model for all data sets is lower than the threshold error which is defined by the designer (Liu 2013; Parsaie and Haghiabi 2015; Parsaie et al. 2016).

2- 4- Support vector regression

SVMs are a set of related supervised learning methods used for classification and regression. In many applications, a non-linear classifier provides a better accuracy. In SVM, the input x is first mapped onto an m-dimensional feature space using some fixed (non-linear) mapping, and then a linear model is constructed in this feature space. The naive way of making a non-linear classifier out of a linear classifier is to map our data from the input space *X* to a feature space *F* using a non-linear function $\varphi: x \rightarrow f$ In the space *F*, the discriminant function is:

$$f(x)=w^T\varphi(x)+b \qquad (11)$$

Using mathematical notation, the linear model (in the feature space) *f*(*x, w*) is given by:

$$w=\sum_{i=1}^{n}\alpha_i x_i \qquad (12)$$

$$f(x,w)=\sum_{j=1}^{n}\alpha_i x_i\varphi_i(x)+b \qquad (13)$$

$$f(x)=\sum_{i=1}^{n}\alpha_i x_i^T x+b \qquad (14)$$

In the feature space, *F*, this expression takes the form:

$$f(x)=\sum_{i=1}^{n}\alpha_i \varphi(x_i)^T \varphi(x)+b \qquad 0\le \alpha_i \le C \tag{15}$$

$$k(x,x')=\varphi(x)^T \varphi(x') \tag{16}$$

$$f(x)=\sum_{i=1}^{n}\alpha_i k(x,x_i)+b \tag{17}$$

There are many kernel functions in SVM, hence how to select a good kernel function is also a research issue. However, for general purposes, there are some popular kernel functions.

I. Linear kernel: $k(x_i, x_j) = x_i^T x_j$

II. Polynomial kernel: $k(x_i, x_j) = (\gamma x_i^T x_j + r)^d$, $\gamma > 0$

III. RBF kernel: $k(x_i, x_j) = exp(-\gamma x_i - x_j)^2$, $\gamma > 0$

IV. Sigmoid kernel: $k(x_i, x_j) = \tanh(\gamma x_i^T x_j + r)$, $\gamma > 0$

Here γ, r and d are kernel parameters. It is well known that SVM generalization performance (estimation accuracy) depends on a good set of meta-parameters parameters γ, r and d are the kernel parameters. The choices γ, r and d control the prediction (regression) model complexity. The problem of optimal parameter selection is further complicated because the SVM model complexity (and hence its generalization performance) depends on all three parameters. Kernel functions are used to change the dimensionality of the input space to perform the classification [20,26,27].

3- Results and Discussion

In this study, to find out the most effective factors on C_d using GT, different scenarios were considered. In each scenario, the effect of one of the input variables was evaluated. Firstly, at scenario number one, all variables were included in GT analysis and in the next scenarios, one of them was removed and again the GT was analyzed. The results of scenarios are given in Table 2. The GT parameters such as gamma, gradient, standard error and V-ratio were chosen as criteria to define the most effective factors. The scenario which had a minimum value for the GT parameters include the most influential parameters on C_d. The variation of the V-ratio is between the 0 and 1. This point is notable that whatever this factor attends to zero shows that related scenario could accurately predict the output.

Reviewing Table 2 shows that scenario number (1) that involves all input variables has a minimum value for the GT parameters. Table 2 shows that removing parameters such as H/P, L_w/L_c, and L_w/W_m causes the Gamma value to increase significantly; therefore, it was found that these parameters are the most important parameters on C_d. The variation of gamma, along the standard error values through the data set based on all input variables are shown in Figure 2. Figure 2 shows that the standard error curve and gamma curve are almost flat after point 180. It means for modeling discharge coefficient regarding the collected data set, qualification of 180 data set (80 percent of all dataset) is enough.

Utilizing the MLPNN model as a common type of soft computing techniques is based on the dataset; therefore, collected datasets were divided into two groups as training and testing. The dimensionless parameters which have presented in Equation 2 were considered as inputs and discharge coefficient was desired as model output. Data selection for the preparation of MLPNN model was carried out using a random approach. Based on the GT results, 80 percent of the total dataset was considered for training and remains (20 percent) were used for testing. Training data sets were used for calibration and testing dataset was considered for the model validation. It is notable that the dedicated data must be a good representative of the entire collected data. This means that the range of training and testing should be close together. Designing of structure of MLPNN model is a trial and error process but the experience of the designer reduces the time to try; however, recommendations of the investigators who conducted similar works are useful. In this paper, the recommendations of Parsaie and Haghiabi (2015) was used. Designing of MLPNN model includes the number of hidden layer(s), the number of the neuron(s) in each hidden layer(s), definition of suitable transfer function for the neurons of hidden layer(s), definition of the suitable transfer function for output layer and learning algorithm. To obtain an optimal structure for the MLPNN model, step by step development of MLPNN structure was considered. In this approach, firstly one hidden layer which includes the number of neurons equal to input features is considered. Then, different transfer functions were tested to find out best transfer function with the best performance. After justifying the transfer function, to increase the accuracy of model, increasing the number of hidden layer or (and) number of neurons may be considered. It is notable that the number of the neurons in the hidden layer

Table 2. Results of gamma test analysis in the absence of one variable

Row	absence	Gammas	Gradient	Standard error	V-ratio
1	-	0.0019	0.1557	0. 0014	0.0075
2	H/P	0.0074	0.2218	0.0013	0.0295
3	L_W/W_{mc}	0.0053	0.2018	0.0016	0.0274
4	L_c/W_c	0.0033	0.2915	0.0027	0.0133
5	L_w/W_c	0.0060	0.2619	0.0015	0.0083
6	w_{mc}/W_c	0.0033	0.1758	0.0011	0.0135

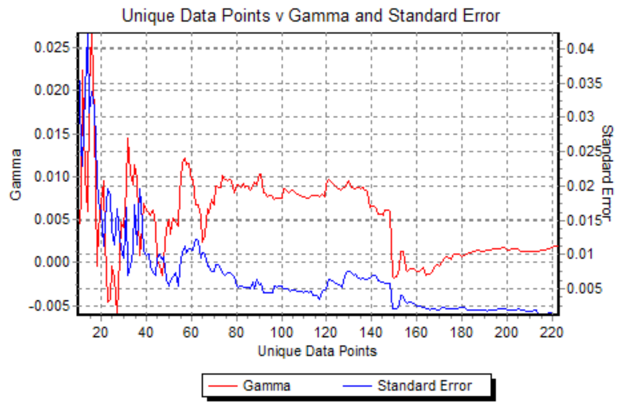


Fig. 2. The variation of Gamma test and standard error with unique data points

is increased one by one. This process continues to obtain a model with a suitable performance. In this study, various transfer functions such as log-sigmoid (logsig), tan-sigmoid (tansig), and linear (purelin) were tested. All stages of preparation of MLPNN were conducted in Matlab software. The developed MLPNN model consists of two hidden layers with five and three neurons existed in the first and second hidden layers, respectively. The tansig and purelin were considered as hidden layers and output layer transfer functions, respectively. During the development of MLPNN model, it was found that adding the number of hidden layers and neurons in the hidden layers has not a significant effect on increasing the model precision and just increases the computation cost. The structure of the developed MLPNN is shown in Figure 3. It is notable that the Levenberg–Marquardt technique was used to learn MLPNN model. The results of MLPNN model in training and testing stages are shown in Figure 4. In this figure, the results of MLPNN model were plotted versus the observed data. Developing the RBFNN was as similar to MLPNN. This means that the same approach was considered. The structure of the developed RBFNN to predict the discharge coefficient is shown in Figure 3. As shown in this figure, the developed RBFNN model includes one hidden layer which consists of eight neurons. The performance of RBFNN is shown in Figure 4. Comparison the results of RBFNN model with MLPNN shows that the accuracy of this is a bit less than the MLPNN model.

Development of support vector machine (SVM) as a powerful soft computing method is based on the dataset. To this end, the same that had used for developing the RBFNN and MLPNN model was used for SVM preparation. As mentioned in the SVM model section, preparation of SVM includes selection of kernel function, arrangement of dataset for training and testing and also choosing the input variable. For this purpose, all of four kernel functions mentioned in the SVM model section were tested. The results of each kernel function are given in Table 3. As presented in this table, the RBF function is more accurate among the kernel functions. To find out the kernel function with the best performance, all the variables were considered as inputs. During the development of SVM model, it was found that the γ=0.3 and epsilon=15.30 for RBF kernel function. The structure of developed SVM model for predicting the C_d is shown in Figure 3. Results of SVM model with RBF as kernel function are shown in Figure 4.

Reviewing Figure 4 indicates that the best performance is related to SVM model among the applied models. This result is based on error indices. Comparing the obtained results in the study conducted by Haghiabi et al. (2017) reveals that developing the ANFIS model based on the results of GT leads to increasing the precision of modeling and prediction. The accuracy of ANFIS is a bit more accurate than the SVM model. This is due to the facility of ANFIS model to assign more weight to parameters which are more effective on output. To present more information for outcomes through the dataset, another index introduced by Noori et al. (2009) that was named developed discrepancy ratio (DDR) was calculated by Equation 18. The results of DDR are shown in Figure 5. Reviewing Figure 5 shows that the best performance is related to SVM model.

$$DDR = \left(\frac{Predicted\ \ Value}{Observed\ \ Value}\right) - 1 \qquad (18)$$

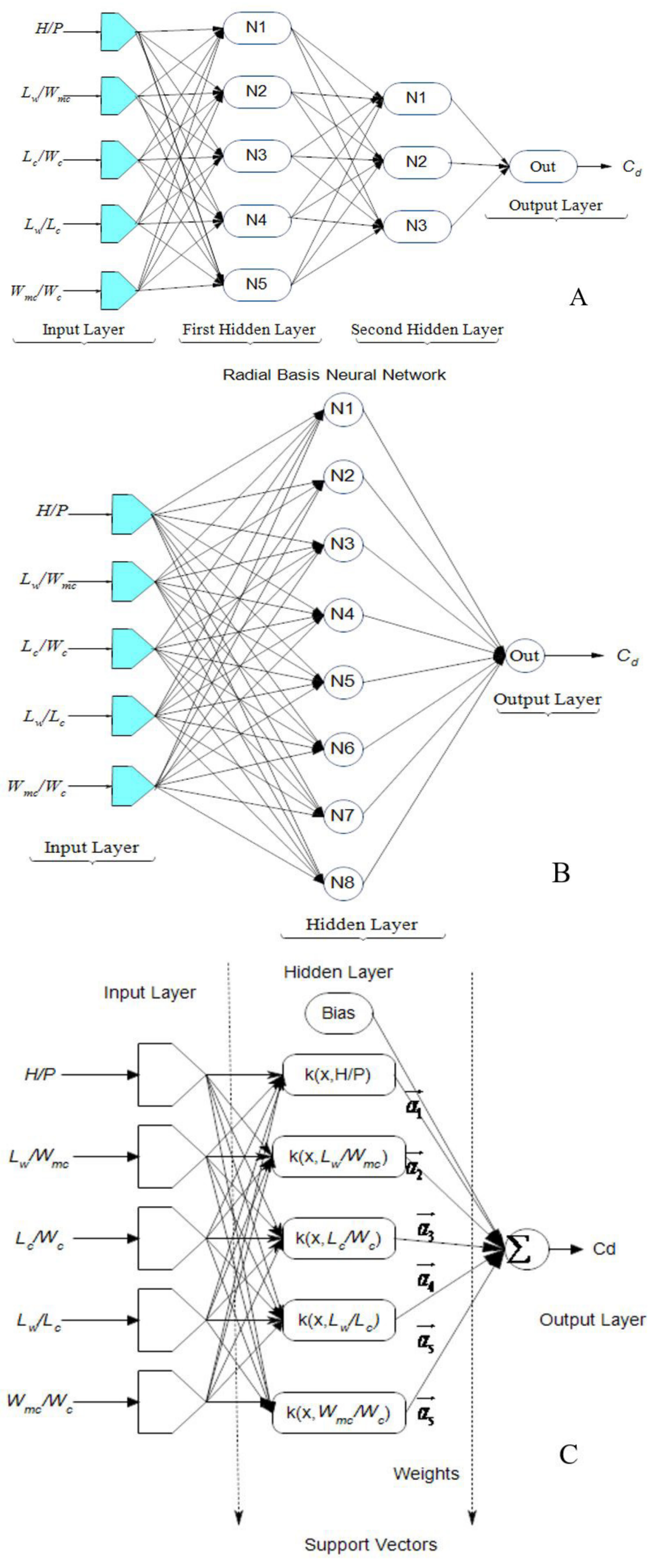


Fig. 3. Structures of developed models for the prediction of C_d of triangular labyrinth weirs (A: MLPNN, B: RBFNN, C: SVM)

As recently stated, the best accuracy among the applied models was related to SVM. Therefore, another scenario for the prediction of C_d is developing the SVM model based on the most effective parameters. It should be noted that the most important parameters were defined previously by GT

Table 3. Summary of SVM results during the development stages

Row	Output	Kernel Function	Training		Testing	
			R^2	RMSE	R^2	RMSE
1	C_d	RBFNN	0.94	0.039	0.97	0.038
2	C_d	Linear	0.73	0.0678	0.65	0.1360
3	C_d	Polynomial	0.86	0.0539	0.76	0.0970
4	C_d	Sigmoid	0.90	0.0480	0.91	0.0794

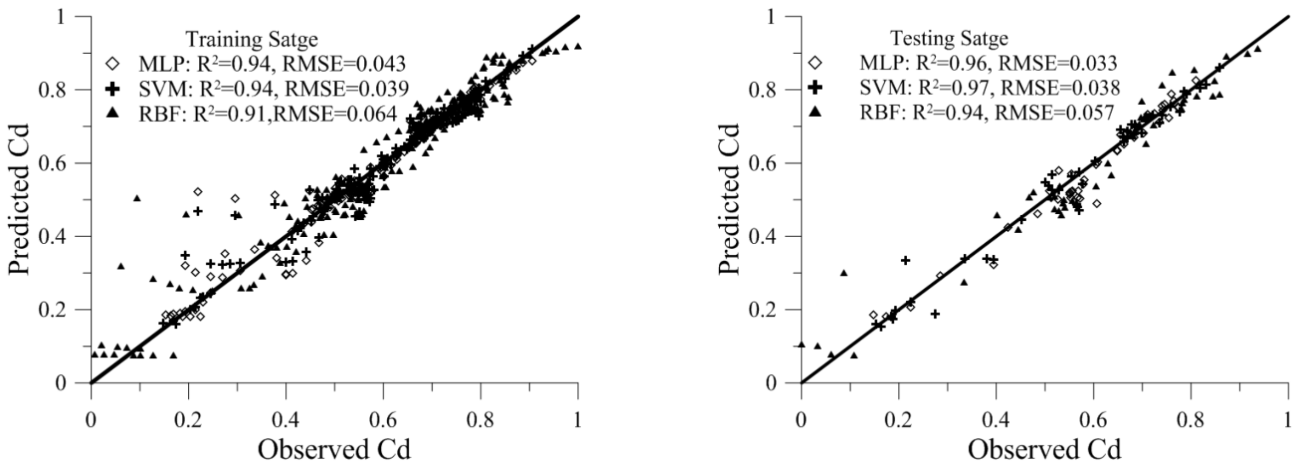


Fig. 4. Results of applied models in training and testing stages

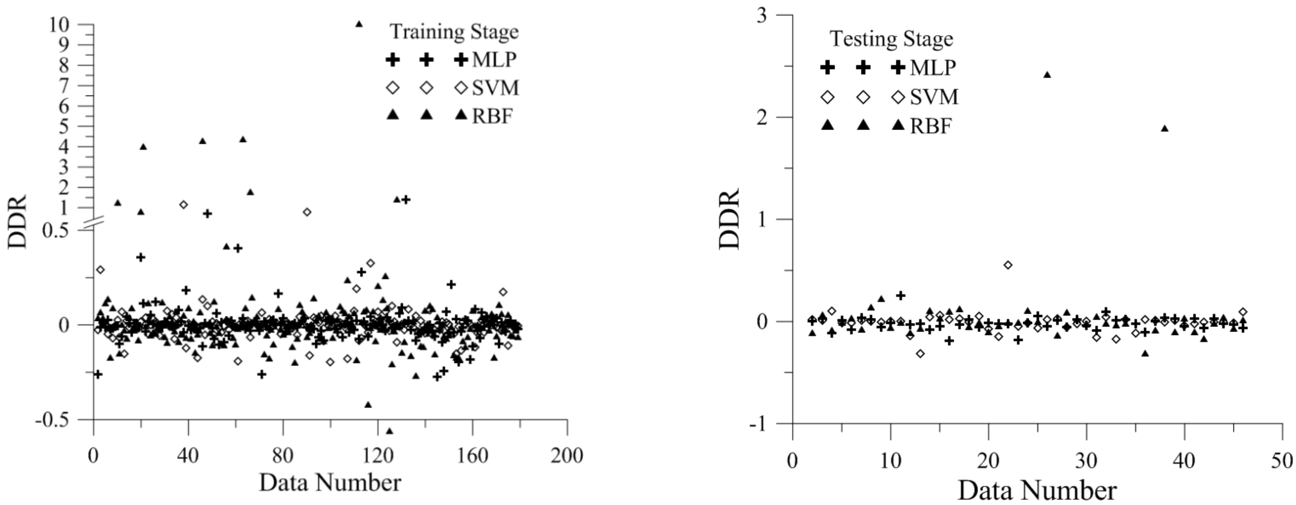


Fig. 5. Results of DDR for outcomes of applied models

analysis. In other words, in a new scenario, the performance of SVM model for the prediction of the C_d with regard to H/P, L_w/L_c, and L_w/W_m as inputs are investigated. Results of predicting the C_d based on the most effective parameters are shown in Figures 10 and 11. As shown in these figures, the accuracy of SVM model based on the most effective parameters was not significantly reduced.

4- Conclusion

Modeling of the hydraulic structure is the main part of hydraulic engineering activities. The estimation of discharge coefficient of hydraulic structure especially weirs is one of the main parameters to design the optimal operational program of hydro-systems. Labyrinth weirs are the novel subjects in the field of hydraulic structure which have been considered as a rational approach for improving the performance of conventional linear weirs. In this paper, the discharge coefficient (C_d) of triangular labyrinth weir was predicted using the multilayer perceptron neural network, radial basis neural network and support vector machine. To this end, the involved parameters were derived using dimensional analysis and related dataset were collected from the literature. To define the most effective parameters on the C_d, the gamma test (GT) was applied. Results of GT declared that H/P, L_w/L_c, and L_w/W_m are the most effective parameters for predicting the C_d. Results of applied models indicated that all the applied models have an acceptable performance; however, the SVM model was more accurate. The SVM model was prepared in two scenarios, first, all the involved parameters were considered as inputs and in the second

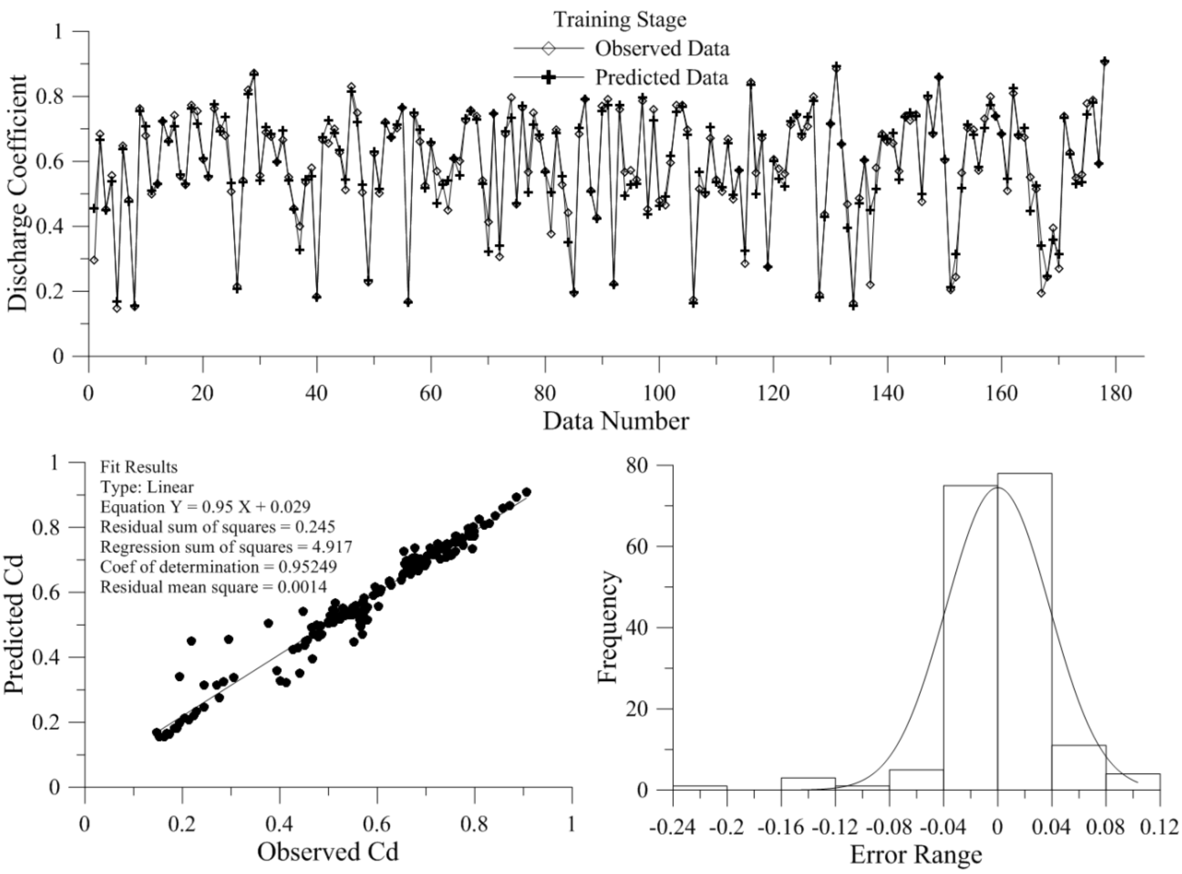


Fig. 6. Results of SVM based on the most effective parameters as inputs

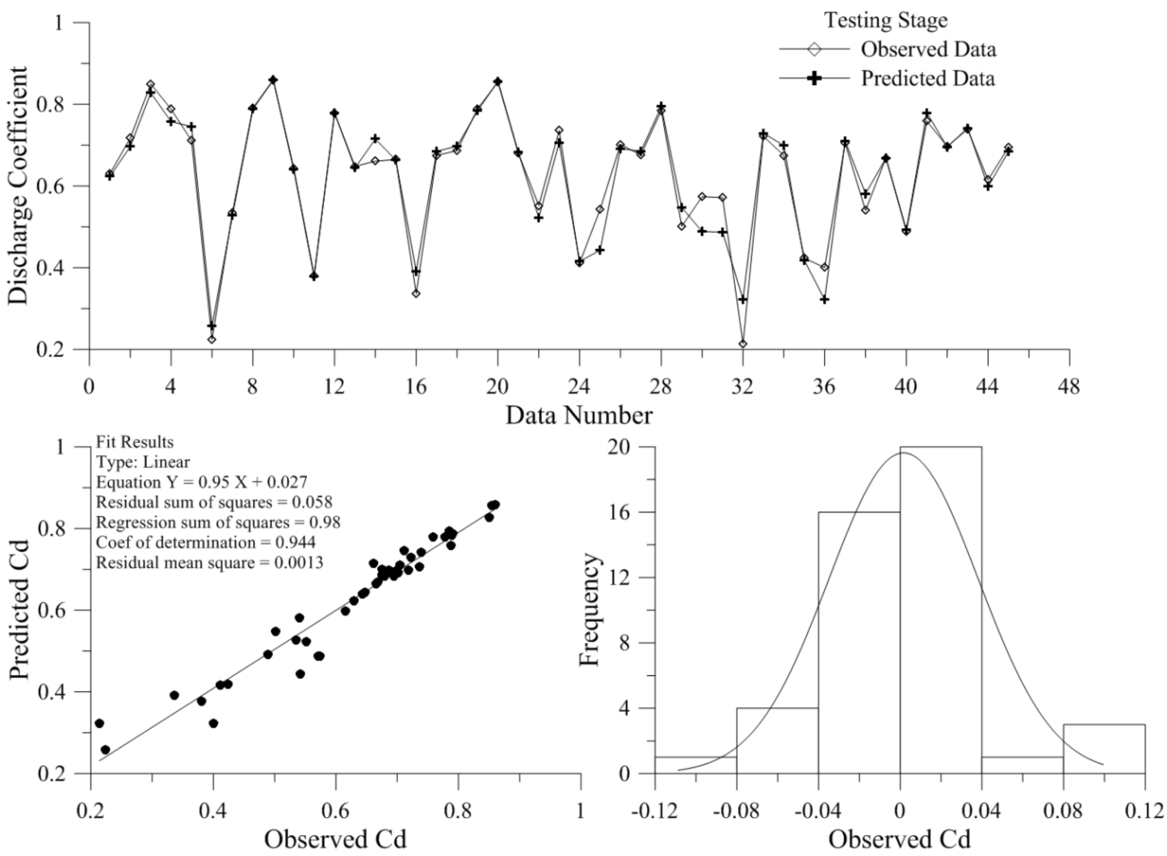


Fig. 7. Results of SVM based on the most effective parameters as inputs

scenario, the SVM model was prepared based on the most effective parameters that had driven from the GT analysis. Comparison of the performance of SVM model in terms of both scenarios shows that preparation of SVM model based on the most effective parameters has no significant effect on decreasing its accuracy.

Notation

ANN's	Artificial Neural Networks
C_d	Discharge Coefficient
CFD	Computational Fluid Dynamic
DDR	Developed Discrepancy Ratio
g	Gravitational Acceleration
GMDH	Group Method Of Data Handling
GP	Genetic Programing
GT	Gamma Test
H	Total Head
L_c	Length Of One Cycle
logsig	Log-Sigmoid
L_w	Total Length Of The Weir
MARS	Multivariate Adaptive Regression Splines
MLPNN	Multilayer Perceptron Neural Network
P	Weir Height
PMF	Probable Maximum Flood
purelin	Linear
R^2	Coefficient Of Determination
RBF	Radial Basis Function
RBFNN	Radial Basis Neural Network
RMSE	Root Mean Square Error
SVM	Support Vector Machine
tansig	Tan-Sigmoid
V	Flow Velocity
W_c	Width Of One Cycle
W_{mc}	Main Channel Width
ρ	Density Of Flow
σ	Surface Tension

References

[1] R. M. Anderson and B. P. Tullis, "Comparison of Piano Key and Rectangular Labyrinth Weir Hydraulics." *Journal of Hydraulic Engineering*, 138(4) (2012) 358-361.

[2] R. M. Anderson and B. P. Tullis, "Piano Key Weir Hydraulics and Labyrinth Weir Comparison." *Journal of Irrigation and Drainage Engineering*, 139(3) (2013) 246-253.

[3] H. M. Azamathulla, A. H. Haghiabi and A. Parsaie, "Prediction of side weir discharge coefficient by support vector machine technique." *Water Science and Technology: Water Supply*, 16(4) (2016) 1002-1016.

[4] S. Dehdar-behbahani and A. Parsaie, "Numerical modeling of flow pattern in dam spillway's guide wall. Case study: Balaroud dam, Iran." *Alexandria Engineering Journal*, 55(1) (2016) 467-473.

[5] M. E. Emiroglu and N. Kaya, "Discharge Coefficient for Trapezoidal Labyrinth Side Weir in Subcritical Flow." *Water Resources Management*, 25(3) (2011) 1037-1058.

[6] S. Erpicum, F. Laugier, J. L. Boillat, M. Pirotton, B. Reverchon and A. Schleiss, "Labyrinth and piano key weirs—PKW 2011." Proc., *Proceedings of the International Conference on Labyrinth and Piano Key Weirs*, Balkema Liege, 9-11.

[7] S. Erpicum, F. Laugier, M. Pfister, M. Pirotton, G. M. Cicero and A. J. Schleiss, . Labyrinth and Piano Key Weirs II, Taylor & Francis (2013).

[8] M. Ghodsian, "Stage–discharge relationship for a triangular labyrinth spillway." *Proceedings of the ICE-Water Management*, 162(3) (2009) 173-178.

[9] A. H. Haghiabi, "*Modeling River Mixing Mechanism Using Data Driven Model*." Water Resour Manage, (2016) 1-14.

[10] A. H. Haghiabi, "Prediction of longitudinal dispersion coefficient using multivariate adaptive regression splines." *Journal of Earth System Science*, 125(5) (2016) 985-995.

[11] A. H. Haghiabi, H. M. Azamathulla and A. Parsaie, "Prediction of head loss on cascade weir using ANN and SVM." *ISH Journal of Hydraulic Engineering*, (2016) 1-9.

[12] A. H. Haghiabi, A. Parsaie, and S. Ememgholizadeh, "Prediction of discharge coefficient of triangular labyrinth weirs using Adaptive Neuro Fuzzy Inference System." *Alexandria Engineering Journal* (2017).

[13] N. Hay and G. Taylor, "Performance and design of labyrinth weirs." *Journal of the Hydraulics Division*, 96(11) (1970) 2337-2357.

[14] A. Hossein Zaji, H. Bonakdari, and S. Karimi, "Radial Basis Neural Network and Particle Swarm Optimization-based equations for predicting the discharge capacity of triangular labyrinth weirs." *Flow Measurement and Instrumentation*, 45 (2015) 341-347.

[15] K. Houston, "Hydraulic model study of Ute Dam labyrinth spillway." Report GR-82-7 August 1982. 47 p, 30 Fig, 2 Tab, 4 Ref, 1 Append.

[16] A. Kabiri-Samani and A. Javaheri, "Discharge coefficients for free and submerged flow over Piano Key weirs." *Journal of Hydraulic Research*, 50(1) (2012) 114-120.

[17] O. Kisi, "Pan evaporation modeling using least square support vector machine, multivariate adaptive regression splines and M5 model tree." *Journal of Hydrology*, 528 (2015) 312-320.

[18] S. Kumar, Z. Ahmad and T. Mansoor, "A new approach to improve the discharging capacity of sharp-crested triangular plan form weirs." *Flow Measurement and Instrumentation*, 22(3) (2011) 175-180.

[19] J. Liu, *Radial Basis Function (RBF) neural network control for mechanical systems: design, analysis and Matlab simulation*, Springer Science & Business Media (2013).

[20] O. Machiels, S. Erpicum, B. J. Dewals, P. Archambeau and M. Pirotton, "Experimental observation of flow characteristics over a Piano Key Weir." *Journal of Hydraulic Research*, 49(3) (2011) 359-366.

[21] O. Machiels, M. Pirotton, A. Pierre, B. Dewals and S. Erpicum, "Experimental parametric study and design of Piano Key Weirs." *Journal of Hydraulic Research*, 52(3) (2014) 326-335.

[22] R. Noori, A. Karbassi, A. Farokhnia, and M. Dehghani, "Predicting the Longitudinal Dispersion Coefficient Using Support Vector Machine and Adaptive Neuro-Fuzzy Inference System Techniques." *Environmental Engineering Science*, 26(10) (2009) 1503-1510.

[23] R. Noori, A. R. Karbassi, A. Moghaddamnia, D. Han, M. H. Zokaei-Ashtiani, A. Farokhnia and M. G. Gousheh, "Assessment of input variables determination on the SVM model performance using PCA, Gamma test, and forward selection techniques for monthly stream flow prediction." *Journal of Hydrology*, 401(3–4) (2011) 177-189.

[24] A. Parsaie, H. M. Azamathulla and A. H. Haghiabi, "Physical and numerical modeling of performance of detention dams." *Journal of Hydrology*, (2017).

[25] A. Parsaie and A. Haghiabi, "The Effect of Predicting Discharge Coefficient by Neural Network on Increasing the Numerical Modeling Accuracy of Flow Over Side Weir." *Water Resources Management*, (2015) 1-13.

[26] A. Parsaie, and A. Haghiabi, "Prediction of Side Weir Discharge Coefficient by Genetic Programming Technique." *Jordan Journal of Civil Engineering*, 11(1) (2017) 132-141.

[27] A. Parsaie and A. H. Haghiabi, "Predicting the longitudinal dispersion coefficient by radial basis function neural network." *Modeling Earth Systems and Environment*, 1(4) (2015) 34.

[28] A. Parsaie, A. H. Haghiabi, and A. Moradinejad, "CFD modeling of flow pattern in spillway's approach channel." *Sustainable Water Resources Management*, 1(3) (2015) 245-251.

[29] A. Parsaie, A. H. Haghiabi, M. Saneie and H. Torabi, "Applications of soft computing techniques for prediction of energy dissipation on stepped spillways." *Neural Computing and Applications*, (2016) 1-17.

[30] A. Parsaie, S. Najafian and Z. Shamsi, "Predictive modeling of discharge of flow in compound open channel using radial basis neural network." *Modeling Earth Systems and Environment*, 2(3) (2016) 150.

[31] M. L. Ribeiro, M. Pfister, A. J. Schleiss and J.-L. Boillat, "Hydraulic design of A-type Piano Key Weirs." *Journal of Hydraulic Research*, 50(4) (2012) 400-408.

[32] G. K. Robertson, "*Labyrinth weir hydraulics: Validation of CFD modelling.*" Stellenbosch: Stellenbosch University, (2014).

[33] G. D. Singhal, N. Sharma and C. S. P. Ojha, "EXPERIMENTAL STUDY OF HYDRAULICALLY EFFICIENT PIANO KEY WEIR CONFIGURATION." *ISH Journal of Hydraulic Engineering*, 17(1) (2011) 18-33.

[34] G. Taylor, "*The performance of labyrinth weirs.*" Ph.D, University of Nottingham, (1968).

Please cite this article using:

A. Parsaie and A. H. Haghiabi, Support Vector Machine to predict the discharge coefficient of Sharp crested w-planform weirs, *AUT J. Civil Eng.*, 1(2) (2017) 195-204.

DOI: 10.22060/ceej.2017.13005.5309