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The evaluation of the Friction Demand Factor in Loop Ramps of Interchange Facilities

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ABSTRACT: The aim of this study is to evaluate the side friction demand factor in the loop ramps of interchange facilities. The substantial exclusivity of these ramps is the existence of horizontal curves, combined with longitudinal grades. In this study, CarSim and TruckSim software packages, as simulation tools, are applied. Both passenger cars and heavy vehicles are used. The vehicles used in simulations were Hatchback and Sedan (as passenger cars) and Truck (as heavy vehicle). In addition, two various types of loop ramps, including Curve-Curve-Curve and Spiral-Curve-Spiral, in two different conditions (braking and no-braking) are examined. The results showed that the side friction demand factor values assumed by AASHTO Green book (as a main geometric design guideline) are uncertain. In the condition of no-braking, the differences between AASHTO values and the simulation results for uphill and downhill states are 24% and 18%, respectively. In braking condition, similar differences for uphill and downhill are 124% and 135%, respectively. Additionally, based on the regression analysis of the simulation results, the appropriate side friction demand factor models were achieved for different conditions. The findings of the study verify the necessity of revising the friction demand values, especially for the design of interchange loop ramps.

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1- Introduction

The side friction factor between tire and pavement is considered as an important safety parameter, which can affect the rate of vehicle crashes [1]. This parameter is defined as the ratio of the horizontal force to the vertical force [2]. The side friction demand factor is the amount of friction coefficient that vehicle needs to stabilize [3]. In order to design the horizontal curves, conventional geometric design guidelines [4] recommended the friction demand factors, by using a simple mathematical model called Point Mass (PM). In this model, the vehicle is represented as a Point Mass (PM). Due to the simplified assumptions of PM model, no consideration is given to the distribution of frictional forces between different tires of a vehicle. Although the application of PM model for the design of the simple horizontal curves can prepare sufficient margin of safety against both skidding and roll over, the model has serious limitations especially for the design of the facilities, which have combinations of the horizontal curve and longitudinal grades. Loop ramps are considered as one of such facilities. In AASHTO Green book, which is based on PM model, the designs for the alignment plan and the profiles are executed in separate procedures. Therefore, the use of AASHTO Green book seems not to be sufficient for the mentioned facilities. To cover the shortages of the PM model, vehicle dynamic models such as multi-body model are developed. These models are regarded as a basis for simulation analysis, which evaluates vehicle stability on 3-D alignments.

The aim of this study is to evaluate the friction demand factor in loop ramps of the interchange facilities. The main contribution of this paper is to determine the required side friction demand factor in loop ramps, through a simulation-based methodology. CarSim and TruckSim software packages, which are based on the multi-body models, are employed as simulation tools. These software packages are able to animate the vehicle performance and plot the diagrams. In addition, we have investigated relationships between the friction demand factor and different parameters used to design the loop ramps of the interchange facilities.

The outline of the paper is as follows: in section 2, the background of vehicle stability models is explained. The methodology is described in section 3. The results of the simulation are presented in section 4. Finally, the concluding remarks are given to summarize the contribution of the paper.

2- Background

Starting in the late 1940s, friction factor studies were established, regarding the driver's comfort [6]. Based on vehicle stability analysis, different methods such as point mass model, bicycle model, and multi-body simulation were presented [5, 7, 8]. Application of each of the mentioned models is related to their easiness and the accuracy required. Brief descriptions of each of the vehicle stability models are

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presented in the following section.

2- 1- Point mass model

The Point Mass (PM) model is the most popular vehicle stability analysis, which has been used in conventional geometric design guidelines. PM model considers a cornering vehicle as a rigid and un-sprung mass in which dimensions of vehicle body do not have any effect on vehicle behavior [9, 10]. PM model is obtained from logical mathematics equations and simulates the vehicle motion with the main assumption that all forces entered a vehicle are concentrated at one point. The main features of this model are the ease of use and consideration of truck as a passenger car [5]. The cornering vehicle in point mass model undergoes centripetal acceleration. Centrifugal force causes the vehicle to skid and roll over and is neutralized by a combination of the side friction force and the weight force contributed by the exertion of super-elevation [11, 12]. The PM model employs the following calculations:

$$a_r = \frac{v^2}{R} \tag{1}$$

$$m.a_r = \sum F_y \tag{2}$$

$$w\sin\alpha + f(w\cos\alpha) = \frac{wv^2}{gR}\cos\alpha \tag{3}$$

$$0.01e + f = \frac{v^2}{gR} \tag{4}$$

$$f = \frac{V^2}{gR} - 0.01e \tag{5}$$

The above parameters are defined as follows: a_r centrifugal acceleration, V velocity, R radius curve, W weigh of the vehicle, α super-elevation angle, f side friction factor between tire and pavement surface, g gravity of earth and e super-elevation.

AASHTO Green book [4] defines equation of point mass, as follows:

$$R_{min} = V^2/(127(f_{max} + e_{max})) \tag{6}$$

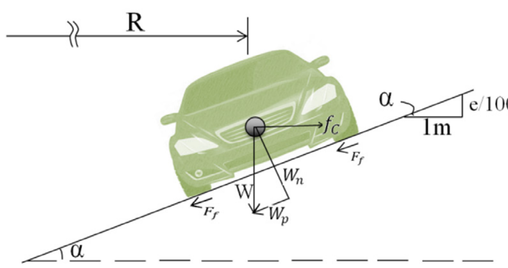


Figure 1. Cornering vehicle forces in PM model

Where f_{max} and e_{max} are the maximum side friction demand factor and the maximum super-elevation, respectively. According to logics of Equation. 6, f_{max} is computed by safety factors and driver comfort. The friction demand is a function of roadway type, road surface conditions, and conditions of tire [5, 8]. In Figure. 2 , the side friction demand factor assumed in AASHTO Green book is illustrated.

According to point mass model, the cornering vehicle will not be able to stabilize if the friction demand exceeds friction supply. In other words, if the friction supply factor is not adequate, the vehicle will be skidding out of the curve [2, 11]. Friction Supply supply factor is the existing friction between tire and pavement surface that causes the vehicle not to skid. It depends on the type and condition of the pavement, vehicle speed, a feature of braking and the type of the tire [6, 13]. If the values of the friction supply and the demand factors are close to each other, skidding margin of safety will decrease. Skidding margin of safety is the difference between the friction demand factor and the friction supply factor [14]. Rolling over the margin of safety is the difference between maximum lateral acceleration that a vehicle is able to withstand without rolling over and current lateral acceleration [11]. Inputs of point mass model consist of roadway geometric features (including radius and super-elevation), velocity and lateral friction factor (f_y). In spite of the vast application, the point mass model has different limitations. PM model does not consider size, dimensions, dynamic parameters of the vehicle, and distribution of friction factors between tires and the vehicle stability on 3-D alignment [3, 5].

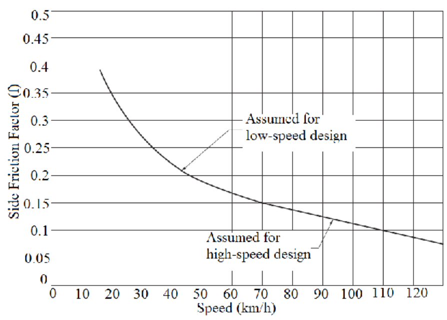


Figure 2. Side friction Demand factor assumed for design [4]

2- 2- Development of dynamic stability models

Unlike the PM model, the vehicle dynamic model paid attention to dynamic aspects of the vehicle stability. Vehicle dynamic behavior and modeling studies were started since 1960s decade at Michigan Transportation Institute [15]. During its evolution process, the bicycle model was presented. This model was introduced by concentrating on different locations of the two tires at a specific distance from the center of the mass. In this model, force, moment equilibrium, and roll equilibrium were introduced, and the vehicle was modeled as a longitudinal axle, in which either front or rear axles, as a single tire, are located at the middle of each axle. Then, with the development of vehicle dynamic model, the vehicle width was added to the bicycle model as an important parameter. In these models, the location of the mass center is regarded above the ground, which allows the weight of vehicle to shift laterally [5, , 7,, 16].

2- 3- Multi-body model

Multi-body model is a type of vehicle dynamic models, in which each tire of a vehicle is modeled as a segregated kinematic body [10]. Although consideration of different dynamic parameters has made this model more complex than the previous ones, it has guaranteed the high accuracy of the model [10]. In this model, the vehicle needs numerical solvers to handle physical and kinematic equations associated with vehicle motion [7]. Due to the complexity of the multi-body model, software packages should be used for model solving. For instance, CarSim and TruckSim are two software packages, which belong to the Mechanical Simulation Corporation (MSC) products. These packages are able to analyze the vehicle motion based on multi-body simulations [15, 17].

2- 4- Past. Past Studies

In the following, some studies on the assessment of side friction factor are reviewed. Kordani and Molan [3] attempted to obtain the correlation of the pavement friction factor and longitudinal grades, for simple horizontal curves and different vehicle types. In their study, the simulation tests were performed by means of both CarSim and TruckSim multi-body simulation software packages. They also recommended formulas for the friction factor, based on regression analysis. However, they did not pay attention to the combination of different curves. In a study performed by Torbic, O'Laughlin, Harwood, Bauer, Bokenkroger, Lucas, Ronchetto, Brennan, Donnell, and Brown [10], the objective was to develop super-elevation criteria for sharp horizontal curves on steep grades. Field survey was undertaken and vehicle dynamic simulations (point mass, bicycle, and multi-body) were performed to investigate the combinations of horizontal curve and vertical grade design criteria. Easa and Dabbour [5], evaluated the effects of vertical alignment on minimum radius requirements using computer simulation. They focused only on trucks, by using vehicle dynamic model (VDM). Mehrara Molan and Abdi Kordani [2] studied the skidding and rollover of vehicles for horizontal curves on longitudinal grades. They did not pay attention to compound horizontal curves. In their study, series of simulation tests were conducted using CarSim and TruckSim. Two types of behavior for the driving system are considered in their simulations: the driver negotiates the curve at constant speed, or he needs to brake while passing downgrades. Mavromatis, Psarianos, Tsekos, Kleioutis and Katsanos [8] analyzed the vehicle motion on sharp horizontal curves combined with steep longitudinal grades. Their study was based on field measurements, by utilizing a FWD C-Class passenger car in both upgrade and downgrade directions of the road. The vehicle was driven in braking mode on adjacent steep grade tangent sections, in order to define the peak friction coefficients. Kordani, Molan and Monajjem [18], attempted to present a relationship between friction demand factor and longitudinal grades located on horizontal curves by using three-dimensional simulation model. They presented a series of models in order to assess these factors based on design speed, longitudinal grade, and vehicle type (Sedan, SUV, and Truck). Kordani, Sabbaghian, and Tavassoli Kallebasti [19] analyzed the influence of coinciding horizontal curves and vertical sag curves on side friction factor and lateral acceleration, by means of simulation tools. The simulations were performed for four different speeds of 60, 80, 100, and 120 km/h, in which all alignments were designed based on the quantities derived from AASHTO Green Book. However, their input parameters exclude some important parameters like super-elevation and different types of the compound curves. The study presented by Donnell, Wood, Himes, and Torbic [6] provided an analysis of the margin of safety in horizontal curve design based on field surveys. In this study, they considered vehicle type, pavement conditions, and vehicle speed distribution as the variables of the problem. There are also several studies like [5, 7, 9, 10, 12, 13, 16, 20-26], which focus on the friction demand analysis.

3- Research Methodology

The methodology used in this study consists of five steps, as illustrated in Figure. 3.

Each of methodology steps has been explained briefly, as follows:

Step 1) Designing loops in CIVIL 3D software:

In this study, two types of loops are were investigated: three-centered loops, which are called Curve-Curve-Curve (CCC) in this paper, as well as Spiral-Curve-Spiral (SCS). These two types have been designed based on AASHTO Green book principles, for designing speeds of 50 and 60 km/h, regarding values 4%, 6%, 8%, 10%, 12% of super-elevation. The loops were designed based on the curve minimum radius method, by means of CIVIL 3D software package.

Step 2) Transferring to the CarSim and TruckSim software packages:

the The designed loops have been considered as inputs for the CarSim and TruckSim multi-body simulation software packages. Different stations of the loop routes were defined and introduced. Note that CarSim contains the models to simulate passenger cars, whereas TruckSim is specific for motion simulation of a heavy vehicle like trucks.

Step 3) Consideration of different input parameters to simulate:

Considered parameters to simulate and corresponding values to any of them are provided in Table 1.

In both CarSim and TruckSim, the maximum side friction factor between wheel and pavement is assumed 0.85. The differences between elevations of the two interconnecting roads of the loops are assumed 4, 5, 6, 7, 8 meters in both uphill and downhill directions.

The vehicles in three categories including Hatchback and Sedan as the passenger cars, in addition to a six-axle truck with 18340 kg loading (as a heavy vehicle) are considered. The main difference between Hatchback and Sedan is in expanding rear part of the vehicle, which causes a difference in their mass center position and suspension. In Figure. 4, the characteristics of passenger cars used in our study are shown. In addition, in Figure. 5, the characteristics of the considered heavy vehicle are provided. Simulation is performed with two prospects: with braking at the start of the loop, and without breaking. In case of braking, anti-lock braking system (ABS) is used. It is assumed that the braking process reduces the speed to 20 km/h (from 80 to 60 km/h and from 70 to 50 km/h). According to Bonneson [2] and Torbic, Donnell, Brennan, Brown, O'Laughlin, and Bauer [11], deceleration rate is considered -0.85 m/s^2.

Step 4) Simulation running:

Simulation results, obtained due to vehicle motion over the designed loops were derived based on 0.025 second passing times. These results include vertical, longitudinal, and lateral forces on the wheel. For any station, the resultant horizontal force on each wheel was calculated by the interaction of longitudinal and lateral forces. The obtained value was divided to a vertical force on the pavement and then, the outcome was considered as side friction demand factor in each station. For each complete vehicle motion, the maximum side friction demand factor between different stations of the loop was considered as the side friction demand factor for safe passing from the loop.

Step 5) simulation result analysis:

The simulation output data were analyzed by using regression analysis, thanks to the SPSS software package.

Table 1. Simulation input parameters

Parameter	Number of states	Descriptions
Vehicle type	3	B-Class Hatchback (1110 kg), E-Class Sedan (1650 kg), Truck (truck cab 4455 kg, trailer 5500 kg)
Elevation difference	10	+4, +5, +6, +7, +8 meters -4, -5, -6, -7, -8 meters
Design type of loop ramp	2	Three-center loop Curve-Curve-Curve (C-C-C), and Spiral-Curve-Spiral (S-C-S)
Super-elevation	5	4%, 6%, 8%, 10%, 12%
Truck carries payload	1	18340 kg
Velocity	4	50 and 60 km/h (for no-braking condition) 80 to 60 km/h and 70 to 50 km/h (for braking condition)

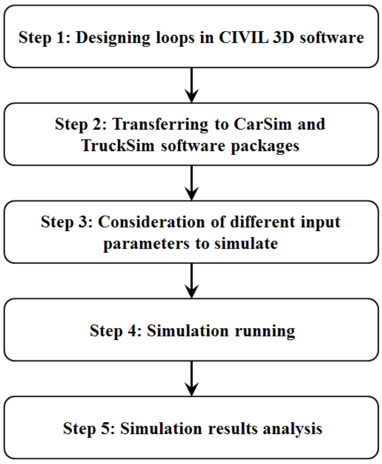
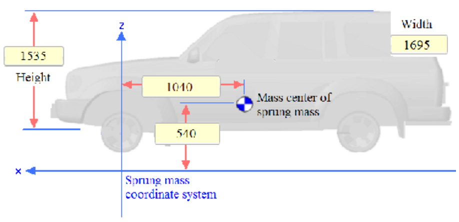
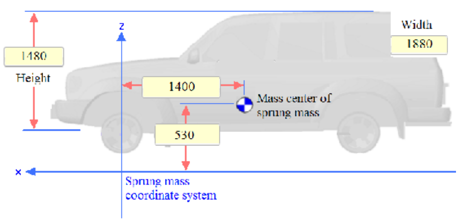


Figure 3. The research methodology

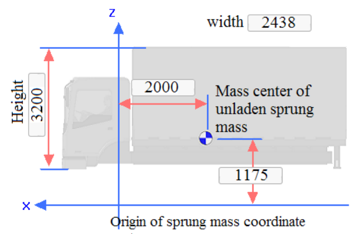


A: B-Class Hatchback

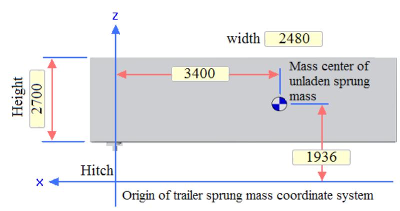


B: E-Class Sedan

Figure 4. Passenger cars used in the simulation (values in millimeters)



A: Truck cab



B: Trailer

Figure 5. Truck used in the simulations (values in millimeters)

4- Simulation results and their analysis

In this study, simulation is performed for 800 various states in CarSim and 400 various states in TruckSim, by changing the input parameters. Some of the simulation results are presented as follows:

4- 1- The effect of different parameters on the side friction demand factor for passenger cars

The simulation outputs for passenger cars in elevation difference of 8 meters in uphill condition are shown in Table 2. Note that in this uphill case, the second interconnecting road is 8 meters higher than first one.

According to Table 2 and other similar simulation data, the following results are attainable:

- For a safe vehicle motion on the loop, the values of friction factor assumed by AASHTO Green book are not adequate. For all simulation cases, those recommended values are less than the required friction factor derived from simulation.
- The friction factor required in front axle of the passenger cars (both Hatchback and Sedan) is more than the corresponding value in the rear axle.
- The braking condition leads to increase in the side friction demand factor, compared with the no-braking condition. This increase is in range of 73% to 136%.
- Design type of the loops is considered as one of the most important parameters, which affects the side friction demand factor.

4- 2- The effect of different parameters on the friction demand factor for heavy vehicles

The truck employed in our simulation tests consists of 6six axles. A scheme of the truck and position of its axles are illustrated in the Figure. 6. .

As shown in Figure. 6, axles 2 and 3 are considered complementary to each other, due to their close locations (couple axles). Additionally, axles 4, 5 and 6 are coupled. The simulation outputs for heavy vehicles in elevation difference of 8 meters in uphill condition are shown in Table 3. Regarding Table 3 and other similar simulation data for heavy vehicles, the following results are achieved:

- The friction demand factor of trucks derived from the simulation is considerably greater than the one assumed by AASHTO.
- In no-braking condition, between couple axles, the maximum share of the side friction demand factor is associated to the rear axle (e.g. axle 3 in couple axles 2, 3, and axle 6 in couple axles 4, 5, 6).

Table 2. Side Friction demand Factor for trucks on upgrade*

Design type of loop	type of vehicle	type of driver behavior	velocity	V= 60 km/h, R=135m & e=4% (SCS) R=260m-135m-260m & e=3.4%-4%-3.4% (CCC) / V= 50 km/h, R=86m & e=4% (SCS) R=170m-86m-170m & e=3.4%-4%-3.4% (CCC)		side friction factor
				axle 1	axle 2	
CCC	Hatchback	braking	70 → 50	0.360	0.238	0.19
CCC	Hatchback	braking	80 → 60	0.361	0.231	0.17
CCC	Hatchback	No-braking	50	0.203	0.183	0.19
CCC	Hatchback	No-braking	60	0.184	0.166	0.17
CCC	Sedan	braking	70 → 50	0.301	0.270	0.19
CCC	Sedan	braking	80 → 60	0.307	0.275	0.17
CCC	Sedan	No-braking	50	0.204	0.180	0.19
CCC	Sedan	No-braking	60	0.185	0.164	0.17
SCS	Hatchback	braking	70 → 50	0.399	0.385	0.19
SCS	Hatchback	braking	80 → 60	0.336	0.323	0.17
SCS	Hatchback	No-braking	50	0.206	0.185	0.19
SCS	Hatchback	No-braking	60	0.187	0.169	0.17
SCS	Sedan	braking	70 → 50	0.399	0.370	0.19
SCS	Sedan	braking	80 → 60	0.338	0.313	0.17
SCS	Sedan	No-braking	50	0.207	0.182	0.19
SCS	Sedan	No-braking	60	0.188	0.167	0.17

* The difference between elevations of the two interconnecting roads of the loop is equal to 8-meters.

Table 3. Side friction demand factor for trucks on upgrade*

Design type of loop	Type of driver behavior	velocity	V=60 km/h, R=135m & e=4% (SCS), R=260m-135m-260m & e=3.4%-4%-3.4% (CCC) / V=50 km/h, R=86m & e=4% (SCS), R=170m-86m-170m & e=3.4%-4%-3.4% (CCC)						side friction factor assumed by AASHTO
			Axle 1	Axle 2	Axle 3	Axle 4	Axle 5	Axle 6	
CCC	braking	70 → 50	0.497	0.626	0.649	0.592	0.567	0.556	0.19
CCC	braking	80 → 60	0.506	0.656	0.670	0.655	0.645	0.623	0.17
CCC	No-braking	50	0.297	0.186	0.288	0.056	0.186	0.307	0.19
CCC	No-braking	60	0.255	0.171	0.247	0.093	0.179	0.260	0.17
SCS	braking	70 → 50	0.473	0.384	0.500	0.322	0.414	0.503	0.19
SCS	braking	80 → 60	0.385	0.348	0.424	0.320	0.383	0.450	0.17
SCS	No-braking	50	0.300	0.232	0.297	0.059	0.186	0.306	0.19
SCS	No-braking	60	0.256	0.176	0.252	0.097	0.184	0.267	0.17

* The difference between elevations of the two interconnecting roads of the loop is equal to 8-meters.

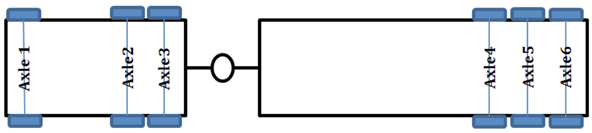


Figure 6. Positions of axles of the truck used for simulation

4- 3- The comparison between passenger cars and heavy vehicles' friction demand factor

Table 4 shows the maximum friction demand factor, for the vehicles moved in the certain height of 8 meters. To sustain the vehicle, all axles should be balanced. Balance conditions of the vehicle will be violated if at least one axle slides. Therefore, in passenger cars, the axle with the maximum friction factor is considered as the critical axle. For a truck, as a heavy vehicle, the sliding of the front axle can unbalance the truck. In addition, as shown in Figure. 6, axles 2 and 3 interact with each other and work together with complementary (located side-by-side asset of a couple of axles). Similarly, axles 4, 5 and 6 are considered a set of coupled axles. For such a set, the axle with more friction demand factor is more probable to slide. However, vehicle skidding occurs whenever the axle with less friction demand factor slides, same as the other axles. On the other hand, the axle with less friction demand factor is counted as the critical one. Therefore, for the set of coupled axles 2 and 3, the minimum friction demand factor (e.g. min (f_2, f_3)) is considered as the basis for maintaining the local sustainability of the rear end of the truck. Similarly, for the set of coupled axles 4, 5 and 6, the minimum friction factor (min (f_4, f_5, f_6)) is critical. Accordingly, the critical friction demand factor of the truck is calculated as follows:

$$f_{Critical}=\max(f_1,(\min(f_2,f_3),\min(f_4,f_5,f_6))) \qquad (7)$$

Regarding Table 4 and other similar tables related to other heights, the following results can be concluded:

- The friction demand factors for heavy vehicles are greater than the ones for passenger cars. This difference is in range of 27% to 52% for no-braking condition and in range of 49% to 62% for braking condition.
- Curve type is one of the effective parameters on the value of the friction demand factor.

4- 4- The effect of height changes on friction demand

In Table 5, the friction demand factors associated with the passenger cars in uphill and downhill are addressed. Regarding Table 5 and other similar tables related to other heights, the following results can be concluded:

- While moving the passenger cars in uphill with constant speed, the greater the difference between elevations of the two interconnecting roads of the loop, the more side friction required to keep sustainability.
- For both types of the passenger cars, the friction demand factors differ in uphill and downhill conditions.
- In braking condition, the vehicle motion in downhill, needs more friction demand in the range of 0.1% to 6.4%, compared with uphill; whereas in no-braking condition, vehicle motion in downhill needs less friction demand in a range of 1.6% to 11.1%, compared with uphill.

4- 5- The comparison of friction demand values: AASHTO and simulation results

The differences of the friction demands derived from simulation and values assumed by AASHTO [4] (Figure. 2) are provided in Table 6. In this table, positive values indicate that the friction factors obtained from simulation are greater. Figure. 7 shows the values of friction demand for the loop ramps with speed design of 60 km/h. Regarding Table 6 and Figure. 7, it is concluded that the maximum difference between the values obtained from simulation and AASHTO occurs in case of the braking condition of the truck. Results show that the AASHTO recommended values for the friction demand factor need to be modified, in order to ensure vehicle safe passing from the loop ramps.

Table 4.: Effects of vehicle type on side friction demand factor*

Design type of loop	type of driver's behavior	velocity	V= 50 km/h, R=86m & e=4% (SCS), R=170-86-170 & e=3.4% - 4% - 3.4% (CCC)			velocity	V= 60 km/h, R=135 & e=4% (SCS), R=260-135-260 & e=3.4%, 4%, 3.4% (CCC)		
			Hatchback	Sedan	Truck		Hatchback	Sedan	Truck
CCC	braking	70 → 50	0.366	0.324	0.676	80 → 60	0.326	0.296	0.691
CCC	No-braking	50	0.196	0.200	0.276	60	0.178	0.182	0.218
SCS	braking	70 → 50	0.417	0.418	0.522	80 → 60	0.346	0.347	0.390
SCS	No-braking	50	0.199	0.203	0.286	60	0.181	0.185	0.226

* The difference between elevations of the two interconnecting roads of the loop is equal to -8 meters.

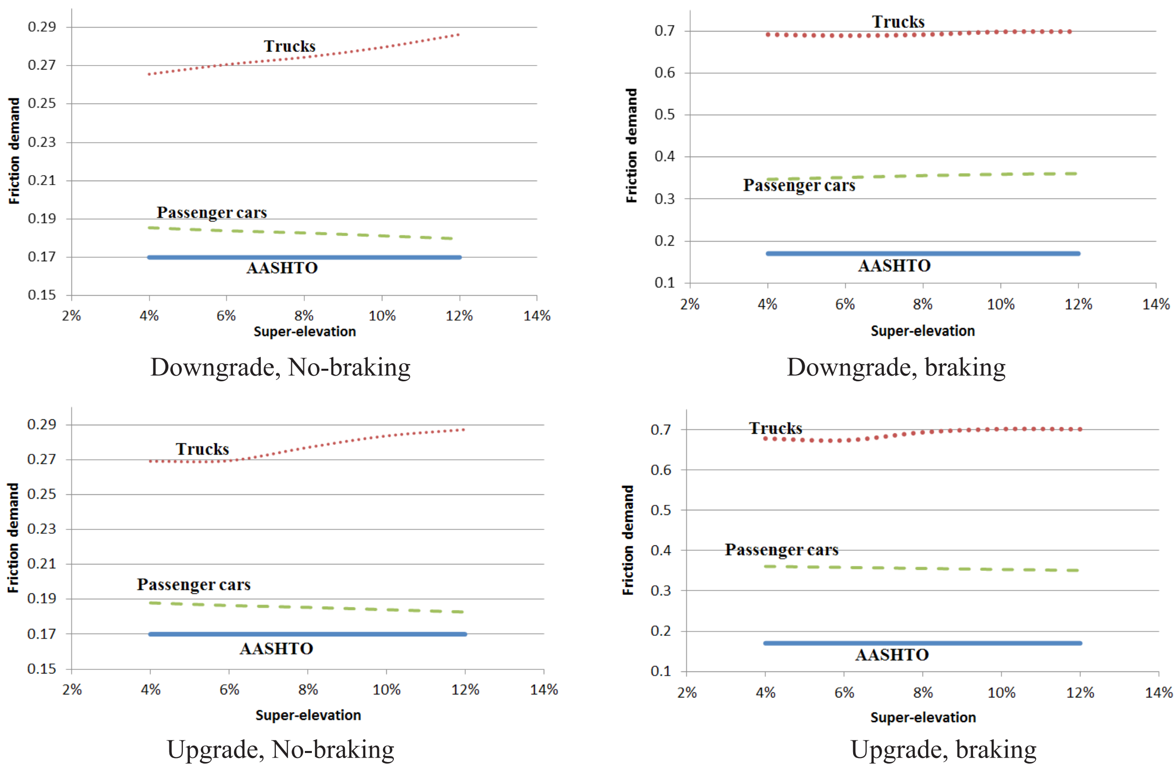


Figure 7. Comparison of friction demand of AASHTO values and simulation results, for passenger cars and trucks at loop speed design of 60 km/h

Table 5. Comparison of passenger car performance in downgrade and upgrade, for types C-C-C, and S-C-S

No-braking (upgrade)						No-braking (downgrade)					
Elevation difference (m)	R=86m,e=4%, hatchback, SCS, V=50 km/h		Elevation difference (m)	R=170m-86m-170m, e=3.4%-4%-3.4% hatch back, CCC, V=50 km/h		Elevation difference (m)	R=86m, e=4%, hatch back, CCC, V=50 km/h		Elevation difference (m)	R=170m-86m-170m, e=3.4%-4%-3.4%, hatch back, CCC, V=50 km/h	
	axle 1	axle 2		axle 1	axle 2		Axle1	Axle2		axle 1	axle 2
4	0.186	0.235	4	0.200	0.184	-4	0.199	0.188	-4	0.196	0.186
5	0.186	0.235	5	0.200	0.184	-5	0.199	0.189	-5	0.196	0.186
6	0.186	0.235	6	0.201	0.184	-6	0.199	0.189	-6	0.196	0.186
7	0.185	0.235	7	0.202	0.183	-7	0.199	0.189	-7	0.196	0.186
8	0.185	0.235	8	0.203	0.183	-8	0.199	0.189	-8	0.196	0.187
	R=86m,e=4%, sedan, SCS, V=50 km/h			R=170m-86m-170m, e=3.4%-4%-3.4% sedan, CCC, V=50 km/h			R=86m, e=4%, sedan, SCS, V=50 km/h			R=170m-86m-170m, e=3.4%-4%-3.4%, sedan, CCC, V=50 km/h	
	axle 1	axle 2		axle 1	axle 2		Axle1	Axle2		axle 1	axle 2
4	0.182	0.234	4	0.203	0.180	-4	0.203	0.183	-4	0.201	0.180
5	0.182	0.234	5	0.203	0.180	-5	0.203	0.183	-5	0.201	0.180
6	0.182	0.234	6	0.204	0.180	-6	0.203	0.183	-6	0.200	0.180
7	0.182	0.234	7	0.204	0.180	-7	0.203	0.184	-7	0.200	0.181
8	0.182	0.234	8	0.204	0.180	-8	0.203	0.184	-8	0.200	0.181
braking (upgrade)						braking (downgrade)					
Elevation difference (m)	R=86m,e=4% hatchback, SCS, V=50 km/h		Elevation difference (m)	R=170m-86m-170m, e=3.4%-4%-3.4%, hatch back, CCC, V=50 km/h		Elevation difference (m)	R=86m,e=4%, e=3.4%-4%-3.4%, hatch back, CCC, V=50 km/h		Elevation difference (m)	R=170m-86m-170m, e=3.4%-4-3%-4%, hatch back, CCC, V=50 km/h	
	axle 1	axle 2		axle 1	axle 2		Axle1	Axle2		axle 1	axle 2
4	0.403	0.391	4	0.346	0.249	-4	0.412	0.403	-4	0.358	0.268
5	0.402	0.389	5	0.350	0.246	-5	0.414	0.405	-5	0.360	0.271
6	0.401	0.388	6	0.353	0.244	-6	0.415	0.406	-6	0.362	0.273
7	0.300	0.386	7	0.357	0.241	-7	0.416	0.408	-7	0.364	0.275
8	0.399	0.385	8	0.360	0.238	-8	0.417	0.410	-8	0.366	0.277
	sedan, SCS, V=50 km/h			sedan, CCC, V=50 km/h			sedan, SCS, V=50 km/h			sedan, CCC, V=50 km/h	
	axle 1	axle 2		axle 1	axle 2		Axle1	Axle2		axle 1	axle 2
4	0.404	0.376	4	0.305	0.277	-4	0.413	0.388	-4	0.318	0.290
5	0.403	0.375	5	0.303	0.276	-5	0.414	0.390	-5	0.320	0.292
6	0.402	0.373	6	0.301	0.274	-6	0.416	0.391	-6	0.321	0.293
7	0.401	0.372	7	0.300	0.272	-7	0.417	0.393	-7	0.323	0.295
8	0.400	0.370	8	0.301	0.270	-8	0.418	0.394	-8	0.324	0.296

Table 6. Difference between side friction demand factors resulting from the simulations and the recommended values of AASHTO

vehicle type	Grade	No-braking		braking	
		50 km/h	60 km/h	50 km/h	60 km/h
Sedan	all grades	5.9%	7.5%	91.9%	89.6%
	upgrade	6.9%	8.2%	85.6%	88.5%
	downgrade	4.9%	6.5%	98.2%	90.6%
Hatch back	all grades	4.1%	5.9%	102.7%	101.2%
	upgrade	5.7%	7.1%	97.4%	104.1%
	downgrade	2.5%	4.1%	107.6%	98.4%
Truck	all grades	54.4%	41.8%	194.6%	212.5%
	upgrade	61.3%	50.2%	188.4%	210.4%
	downgrade	47.4%	33.5%	200.8%	214.7%
Passenger cars	all grades	5.0%	6.5%	97.3%	95.4%
	upgrade	6.3%	7.6%	91.8%	96.4%
	downgrade	3.7%	5.4%	102.9%	94.5%
All vehicles	all grades	21.5%	18.3%	129.7%	134.5%
	upgrade	24.6%	21.8%	124.0%	134.4%
	downgrade	18.3%	14.8%	135.5%	134.6%

5- Regression analysis

As shown in the previous section, the change in simulation input parameters cancould substantially change the friction demand factor. It was also observed that in some circumstances, the AASHTO assumed values for the friction demand factor are less than the required ones. Therefore, in the present study, we attempt to provide the practical models by using regression analysis, in order to determine the friction demand factor. The simulator inputs are considered as model parameters. In order to analyze the data, SPSS 24 is used. Data analysis is performed based on five different categories of the vehicles: Hatchback, Sedan, heavy vehicle (truck), passenger vehicles, and all three types of the vehicles. In the process of doing the analysis, first-step correlation analysis has been done to explore the variables correlated with the demand friction factor and due to the fact that all the independent variables should be independent. At second-step, the variables correlated with the demand friction factor were candidates to enter the equation as independent variables (according to this point of view, all the independent variables should be independent; table Table 7 is an example of correlation analysis). At third-step, by regression analysis the data were analyzed. In the process of regression analysis, the variables correlated with the demand friction factor were entered into the equation as independent variables, then the significant variables were maintained in the model and the other variables were deleted from the model by the trial and error method. At last, the model that had the maximum numerical R-squared index (R^2) was selected as the best model. In these models, the axle's friction demand factor is considered as the dependent variable. All variables used in regression analysis are explained in Table 8. In Table 8, the value f_{AASHTO} comes from Figure. 2. Since in this study, the value R_{min} is calculated based on the AASHTO assumed values, the value f_{AASHTO} can also be calculated via Eq. 8:

$$f_{AASHTO} = \frac{V^2}{127 R_{min}} - e_{max} \tag{8}$$

The regression models are conducted for each of the mentioned five categories. For example, the results of regression analysis for Hatchback category are provided in Table 9.

In Table 9, if the error of a parameter is less than 0.05 (i.e. index t is more than 1.96), that parameter enters the regression model. The parameter coefficients are also provided in column B. The friction demand models obtained from regression analysis are provided as follows (statistical tests or judgments are provided in Table 9 to table 15Table 15):

Hatchback's front axle:

$$f_{demand}=-0.096+V^2/(78.01R_{min})-1.628e_{max}-0.016\,LT+0.003\,Elv-0.011g+0.206|b_a| \tag{9}$$

Hatchback's rear axle:

$$f_{demand}=V^2/(112.49R_{min})-1.129e_{max}-0.055LT-0.004\,Elv-0.005g+0.02|g|+0.162|b_a| \tag{10}$$

Sedans' front axle:

$$f_{demand}=-0.078+V^2/(79.08R_{min})-1.606e_{max}-0.037\,LT-0.002g+0.178|b_a| \tag{11}$$

Sedans' rear axle:

$$f_{demand}=-0.088+V^2/(81.15R_{min})-1.565e_{max}-0.04\,LT+0.002\,Elv-0.011g+0.168|b_a| \tag{12}$$

Passenger cars' front axle:

$$f_{demand}=-0.065+V^2/(78.54R_{min})-1.617e_{max}-1.565\times10^{-5}(m)-0.026\ LT+0.002\ Elv-0.01g+0.192|b_a| \quad (13)$$

Passenger cars' rear axle:

$$f_{demand}=-0.099+V^2/(77.16R_{min})-1.646e_{max}-0.048LT-0.004g+0.165|b_a| \quad (14)$$

First trucks' axle:

$$f_{demand}=V^2/(99.14R_{min})-1.281e_{max}+0.043LT+0.003\ Elv-0.008\ |Elv|-0.012g+0.048|g|+0.248|b_a| \quad (15)$$

Second trucks' axle:

$$f_{demand}=V^2/(224.78R_{min})-0.565e_{max}+0.129\ LT+0.413|b_a| \quad (16)$$

Third trucks' axle:

$$f_{demand}=V^2/(111.60R_{min})-1.138e_{max}+0.088LT-0.004g+0.023|g|+0.342|b_a| \quad (17)$$

Fourth trucks' axle:

$$f_{demand}=-V^2/(1165.18R_{min})+0.109e_{max}+0.167\ LT+0.49|b_a| \quad (18)$$

Fifth trucks' axle:

$$f_{demand}=V^2/(182.73R_{min})-0.695e_{max}+0.119\ LT+0.385|b_a| \quad (19)$$

Sixth trucks' axle:

$$f_{demand}=V^2/(85.35R_{min})-1.488e_{max}+0.069\ LT+0.293|b_a| \quad (20)$$

Required friction demand factor to maintain stability of passenger cars:

$$f_{demand}=V^2/(103.34R_{min})-1.229e_{max}-1.783\times10^{-5}(m)-0.026\ LT+0.006|g|+0.192|b_a| \quad (21)$$

Required friction demand factor to maintain stability of truck:

$$f_{demand}=V^2/(122.94R_{min})-1.033e_{max}+0.107\ LT+0.022|g|+0.327|b_a| \quad (22)$$

Required friction demand factor to maintain stability of all types of vehicles:

$$f_{demand}=V^2/(276.69R_{min})-0.459e_{max}+4.192\times10^{-5}(m)+0.018\ LT+0.019|g|+0.236|b_a| \quad (23)$$

Table 7. Correlation analysis to maintain stability of all types of vehicles (SPSS outputs)

		f_{AASHTO}	m	LT	|g|	|b_a|
f_{AASHTO}	Pearson Correlation	1	.000	.000	.605**	.000
	Sig. (2-tailed)		1.000	1.000	.000	1.000
	N	1200	1200	1200	1200	1200
m	Pearson Correlation	.000	1	.000	.000	.000
	Sig. (2-tailed)	1.000		1.000	1.000	1.000
	N	1200	1200	1200	1200	1200
LT	Pearson Correlation	.000	.000	1	-.057*	.000
	Sig. (2-tailed)	1.000	1.000		.047	1.000
	N	1200	1200	1200	1200	1200
|g|	Pearson Correlation	.605**	.000	-.057*	1	.000
	Sig. (2-tailed)	.000	1.000	.047		1.000
	N	1200	1200	1200	1200	1200
|b_a|	Pearson Correlation	.000	.000	.000	.000	1
	Sig. (2-tailed)	1.000	1.000	1.000	1.000	
	N	1200	1200	1200	1200	1200

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

Table 8. Variables used in regression analysis

Variable	description
f_{demand}	Side friction demand factor for safe passing through loop
f_{AASHTO}	Side friction demand factor recommended by AASHTO
LT	1 if loop type is C-C-C, and 0 if S-C-S
Elv	Difference between elevations of two interconnecting roads of loop
g	Slope of the loop
b_a	Brake deceleration rate
m	Mass of the vehicle
V	Design speed of the loop
R_{min}	Minimum radius of the loop
e_{max}	Maximum super-elevation of the loop

Table 9. Results of Statistical Analysis for Hatchback vehicle by SPSS Software

Dependent variable is the friction demand factor of axle 1				Dependent variable is the friction demand factor of axle 2			
variable	B	t	sig	variable	B	t	Sig
constant	-0.096	-6.630	0.000	$V^2/(127R_{min})-e_{max}$	1.129	28.768	0.000
$V^2/(127R_{min})-e_{max}$	1.628	20.329	0.000	LT	-0.055	-17.552	0.000
LT	-0.016	-10.412	0.000	Elv	-0.004	-3.167	0.002
Elv	0.003	4.629	0.000	g	-0.005	-4.580	0.000
g	-0.011	-4.625	0.000	|g|	0.020	3.888	0.000
$\|b_a\|$	0.206	115.605	0.000	$\|b_a\|$	0.162	44.629	0.000
R^2=0.972				R^2=0.985			

Table 10. Results of Statistical Analysis for Sedan vehicle by SPSS Software

Dependent variable is the friction demand factor of axle 1				Dependent variable is the friction demand factor of axle 2			
variable	B	t	sig	variable	B	t	Sig
constant	-0.078	-3.695	0.000	constant	-0.088	-3.963	0.000
$V^2/(127R_{min})-e_{max}$	1.606	13.710	0.000	$V^2/(127R_{min})-e_{max}$	1.556	12.579	0.000
LT	-0.037	-16.552	0.000	LT	-0.040	-17.261	0.002
g	-0.002	-2.969	0.003	Elv	0.002	2.121	0.035
$\|b_a\|$	0.178	68.435	0.000	g	-0.011	-2.774	0.006
				$\|b_a\|$	0.168	61.116	0.000
R^2=0.929				R^2=0.914			

Table 11. Results of Statistical Analysis for Passenger cars vehicle by SPSS Software

Dependent variable is the friction demand factor of axle 1				Dependent variable is the friction demand factor of axle 2			
variable	B	t	sig	variable	B	t	Sig
constant	-0.065	-4.555	0.000	constant	-0.099	-5.276	0.000
$V^2/(127R_{min})-e_{max}$	1.617	21.071	0.000	$V^2/(127R_{min})-e_{max}$	1.646	15.871	0.000
m	-1.568E-5	-5.837	0.000	LT	-0.048	-24.448	0.000
LT	-0.026	-18.065	0.000	g	-0.004	-5.510	0.000
ELV	0.002	3.838	0.000	$\|b_a\|$	0.165	71.569	0.000
g	-0.010	-4.301	0.000				
$\|b_a\|$	0.192	112.549	0.000				
R^2=0.944				R^2=0.914			

Table 12. Results of Statistical Analysis for each of Trucks' axles by SPSS Software

Dependent variable is the friction demand factor of axle 1				Dependent variable is the friction demand factor of axle 2			
variable	B	t	sig	variable	B	t	Sig
constant	1.281	33.111	0.000	$V^2/(127R_{min})-e_{max}$	0.565	16.002	0.000
$V^2/(127R_{min})-e_{max}$	0.043	14.015	0.000	LT	0.129	17.560	0.000
LT	0.003	2.401	0.017	$\|b_a\|$	0.413	47.794	0.000
ELV	-0.008	-5.711	0.000				
|ELV|	-0.012	-2.408	0.017				
g	0.048	9.471	0.000				
|g|	0.248	69.399	0.000				
$\|b_a\|$	1.281	33.111	0.000				
R^2=0.994				R^2=0.966			
Dependent variable is the friction demand factor of axle 3				Dependent variable is the friction demand factor of axle 4			
$V^2/(127R_{min})-e_{max}$	1.138	21.003	0.000	$V^2/(127R_{min})-e_{max}$	-0.109	-2.577	0.010
LT	0.088	17.422	0.000	LT	0.167	18.971	0.000
g	-0.004	-2.128	0.034	$\|b_a\|$	0.490	47.377	0.000
|g|	0.023	3.706	0.000				
$\|b_a\|$	0.342	57.884	0.000				
R^2=0.988				R^2=0.943			
Dependent variable is the friction demand factor of axle 5				Dependent variable is the friction demand factor of axle 6			
$V^2/(127R_{min})-e_{max}$	0.695	22.067	0.000	$V^2/(127R_{min})-e_{max}$	1.488	64.962	0.000
LT	0.119	18.220	0.000	LT	0.069	14.385	0.000
$\|b_a\|$	0.385	49.926	0.000	$\|b_a\|$	0.293	52.268	0.000
R^2=0.973				R^2=0.883			

Table 13. Results of Statistical Analysis for maintain stability of passenger cars by SPSS Software

Dependent variable			
variable	B	t	sig
$V^2/(127R_{min})-e_{max}$	1.229	45.095	0.000
m	-1.783E-5	-6.691	0.000
LT	-0.026	-17.646	0.000
|g|	0.006	3.034	0.002
$\|b_a\|$	0.192	110.42	0.000
R^2=0.995			

Table 14. Results of Statistical Analysis for maintain stability of truck by SPSS Software

Dependent variable			
variable	B	t	sig
$V^2/(127R_{min})-e_{max}$	1.033	15.664	0.000
LT	0.107	17.454	0.000
|g|	0.022	2.832	0.005
$\|b_a\|$	0.327	45.475	0.000
R^2=0.981			

Table 15. Results of Statistical Analysis for maintain stability of all types of vehicles by SPSS Software

Dependent variable			
variable	B	t	sig
$V^2/(127R_{min})-e_{max}$	0.459	11.348	0.000
m	4.192E-5	36.138	0.000
LT	0.018	5.300	0.000
|g|	0.019	4.322	0.000
$\|b_a\|$	0.236	58.929	0.000
R^2=0.971			

In Equations 9 to 14, the friction demand factor of each front and rear axles (dependent variables) of the passenger cars are examined separately. In Equations 13 and 14, the friction demand factor of each rear and front axle of the passenger cars are checked by combining the Sedan and Hatchback data. In Equations 15 to 20, the friction demand (dependent variable) of each of six axles of truck is examined separately. Here, the vehicle sustainability is defined as passing from the loop ramp without deviation from its route. On the other hand, in case of one axle skidding, the complete body of the vehicle goes unbalanced. In Equations 21 to 23, all the vehicle axles should be balanced in order to maintain sustainability of the vehicle. In Equation 21, it is assumed that the axle which needs more friction demand factor is considered as the critical one, from vehicle skidding point of view. Then this axle can cause vehicle deviation. Therefore, to acquire Equation 21, the axle with more friction demand factor is regarded as the one for which the dependent variable is calculated. Equation 22 is related to the truck in the friction demand factor model. In the procedure of finding this model, the assumptions mentioned to calculate the minimum friction factor based on sets of coupled axles are respected. Finally, in Equation 23, the friction demand model associated with the data of all vehicles (passenger and heavy vehicles) is provided.

6- Conclusions

The aim of this study is to evaluate the friction demand factor in loop ramp. This type of turning roadway facilities is commonly used in interchanges. The substantial exclusivity of these types of the ramps is the existence of the horizontal curves, combined with the longitudinal grades. In the proposed methodology of this paper, the CarSim and TruckSim software packages are employed, as the simulation tools. They are able to animate the vehicle performance and plot the diagrams. Several parameters are considered as the inputs of the simulation. The vehicles used in simulations are Hatchback and Sedan (as passenger cars) and Truck (as heavy vehicle). Simulation is performed for two various types of loops, including Curve-Curve-Curve (C-C-C) and Spiral-Curve-Spiral (S-C-S), in two different conditions (braking, and no-braking). The results of the study show that the friction demand values recommended by AASHTO (as geometric design guideline) are uncertain. In other words, to ensure safe vehicle passing from loops, the assumed friction factors by AASHTO are lower than the required friction factor (derived from simulation). For no-braking condition, these differences in uphill and downhill states are 24% and 18%, respectively. For braking condition, the difference between simulation results and the AASHTO values, in uphill and downhill are 124% and 135%, respectively. The main reason of these considerable differences may be the type of the models used in AASHTO and the simulation process. In the AASHTO guideline, the Point Mass (PM) model is used for the friction demand calculation. In the PM model, parameters related to the vehicle, curve type, elevation difference, uphill/downhill states in moving directions, braking deceleration, etc. are ignored. While, in our study, simulation is performed based on a multi-body model, and all mentioned parameters are considered in the simulation process. According to other results, friction factor required in front axle of passenger cars is more than the corresponding value in their rear axle. In addition, the simulation verified the fact that the braking condition results in the increase of the friction demand factor. This increase is in the range of 73% to 136% (compared to the condition of no-braking). Furthermore, heavy vehicles' friction demand is more than the one related to passenger cars. This difference is in the range of 27% to 52% for the condition of no-braking and in the range of 49% to 62% for the condition of braking. Based on the regression analysis of the simulation results, the friction demand factor models are achieved for different conditions. The findings of the study verify the necessity of the revising friction demand values, especially for the design of interchange loop ramps.

References

[1] S. Najafi, G.W. Flintsch, A. Medina, Linking roadway

crashes and tire–pavement friction: a case study, International Journal of Pavement Engineering, 18(2) (2017) 119-127.

[2] A. Mehrara Molan, A. Abdi Kordani, Multi-Body Simulation Modeling of Vehicle Skidding and Roll over for Horizontal Curves on Longitudinal Grades, in: Transportation Research Board 93rd Annual Meeting, 2014.

[3] A.A. Kordani, A.M. Molan, The effect of combined horizontal curve and longitudinal grade on side friction factors, KSCE Journal of Civil Engineering, 19(1) (2015) 303-310.

[4] A.A.o.S. Highway, T. Officials, A Policy on Geometric Design of Highways and Streets, 2011, Aashto, 2011.

[5] S.M. Easa, E. Dabbour, Design radius requirements for simple horizontal curves on three-dimensional alignments, Canadian Journal of Civil Engineering, 30(6) (2003) 1022-1033.

[6] E. Donnell, J. Wood, S. Himes, D. Torbic, Use of Side Friction in Horizontal Curve Design: A Margin of Safety Assessment, Transportation Research Record: Journal of the Transportation Research Board, (2588) (2016) 61-70.

[7] T. Varunjikar, Design of horizontal curves with downgrades using low-order vehicle dynamics models, The Pennsylvania State University, 2011.

[8] S. Mavromatis, B. Psarianos, P. Tsekos, G. Kleioutis, E. Katsanos, Investigation of vehicle motion on sharp horizontal curves combined with steep longitudinal grades, Transportation Letters, 8(4) (2016) 220-228.

[9] T.-H. Chang, Effect of vehicles' suspension on highway horizontal curve design, Journal of transportation engineering, 127(1) (2001) 89-91.

[10] D. Torbic, M. O'Laughlin, D. Harwood, K. Bauer, C. Bokenkroger, L. Lucas, J. Ronchetto, S. Brennan, E. Donnell, A. Brown, NCHRP Report 774: Superelevation Criteria for Sharp Horizontal Curves on Steep Grades, Transportation Research Board of the National Academies, Washington, DC, (2014).

[11] D. Torbic, E. Donnell, S. Brennan, A. Brown, M. O'Laughlin, K. Bauer, Superelevation design for sharp horizontal curves on steep grades, Transportation Research Record: Journal of the Transportation Research Board, (2436) (2014) 81-91.

[12] F. Awadallah, Theoretical analysis for horizontal curves based on actual discomfort speed, Journal of transportation engineering, 131(11) (2005) 843-850.

[13] J. Hall, K. Smith, L. Titus-Glover, J. Wambold, T. Yager, Z. Rado, Guide for pavement friction. NCHRP. Web-only document 108. Contractor's inal Report NCHRP Project 01-43, Transportation Research Board of the National Academes, (2009).

[14] J. Morrall, R. Talarico, Side friction demanded and margins of safety on horizontal curves, Transportation Research Record, 1435 (1994) 145.

[15] http://www.carsim.com, Mechanical Simulation Corporation, in, 2017.

[16] G. Furtado, S. Easa, A. Abd El Halim, Vehicle stability on combined horizontal and vertical alignments, National Library of Canada= Bibliothèque nationale du Canada, 2003.

[17] M.W. Sayers, Vehicle dynamics programs for roadway and roadside studies, UMTRI-98-20-1, Transportation Research Institute, University of Michigan, 1999.

[18] A.A. Kordani, A.M. Molan, S. Monajjem, New Formulas of Side Friction Factor Based on Three-Dimensional Model in Horizontal Curves for Various Vehicles, in: T&DI Congress 2014: Planes, Trains, and Automobiles, 2014, pp. 592-601.

[19] A.A. Kordani, M.H. Sabbaghian, B. Tavassoli Kallebasti, Analyzing the influence of Coinciding Horizontal Curves and Vertical Sag Curves on Side Friction Factor and Lateral Acceleration Using Simulation Modeling, TRR Journal, (2015).

[20] J. Bonneson, NCHRP Report 439: Superelevation Distribution Methods and Transition Designs, Transportation Research Board, National Research Council, Washington, DC, (2000).

[21] J.A. Bonneson, Side friction and speed as controls for horizontal curve design, Journal of Transportation engineering, 125(6) (1999) 473-480.

[22] S. Easa, A. El Halim, Radius requirements for trucks on three-dimensional reverse horizontal curves with intermediate tangents, Transportation Research Record: Journal of the Transportation Research Board, (1961) (2006) 83-93.

[23] J. Gattis, B. Finley Vinson III, L.K. Duncan, Low-speed horizontal curve friction factors, Journal of transportation engineering, 131(2) (2005) 112-119.

[24] S. Mavromatis, E. Papadimitriou, B. Psarianos, G. Yannis, Vehicle Skidding Assessment through Maximum-Attainable Constant-Speed Investigation, Journal of Transportation Engineering, Part A: Systems, 143(9) (2017) 04017044.

[25] H.F. Mollashahi, K. Khajavi, A.K. Ghaeini, Safety Evaluation and Adjustment of Superelevation Design Guides for Horizontal Curves Based on Reliability Analysis, Journal of Transportation Engineering, Part A: Systems, 143(6) (2017) 04017013.

[26] K. You, L. Sun, Reliability analysis of vehicle stability on combined horizontal and vertical alignments: Driving safety perspective, Journal of Transportation Engineering, 139(8) (2013) 804-813.

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