

AUT Journal of Civil Engineering

AUT J. Civil Eng., 3(2) (2019) 213-224 DOI: 10.22060/ajce.2019.15254.5528



An Evolutionary Algorithm for Deriving Optimal Operating Policy Under Uncertainties for Tehran Multi-reservoir System

F. Karami*, A. B. Dariane

Department of Civil Engineering, Khajeh Nasir Toosi University of Technology, Tehran, Iran

ABSTRACT: Projecting future inflows under climate change and rapidly growing population has large uncertainty and requires serious attention for proper utilization of limited water resources. Existing algorithms can only optimize the operation policy for a specified scenario (e.g., drought, wet, or normal year; decreased or increased demands) and when established, the system would face serious operational difficulty if the expected scenario does not occur. On the other hand, most of water resources systems involve more than three objectives and demand proper techniques to handle computational complexities in so-called many-objective problems. This paper aims at providing a many-objective optimization algorithm using social choice (SC) and melody search (MeS) algorithms that is able to efficiently derive general system operation rules suitable for all possible future scenarios. In other words, the proposed algorithm overcomes uncertainties in the occurrence of future scenarios and works optimally regardless of future conditions; whether it be variable streamflows and/or increased water demands. To evaluate the performance of the proposed algorithm, a system consisting of five reservoirs in the Tehran region with four objective functions. It is shown that in all cases the general multi-scenario rule derived by the proposed method performs as good as each of the operation rules derived for every specific scenario assuming the occurrence of that scenario. Moreover, the proposed many-objective algorithm is able to handle as many objectives as needed without any computational burden and/or algorithm complexity.

1- Introduction

Multi-reservoir multi-objective system operation is a complex decision-making issue that involves many variables, objectives and considerable risk and uncertainty. In order to maximize benefits or minimize costs and losses, reservoir operators usually prefer to follow specified rule curves which determine the actions that should be taken based on the current state of the system. Rule curves are typically determined by a combination of simulation and optimization techniques. However, uncertainties around the supply and demand side are difficulties for water resources planning and management [1]. Significant variations in inflows have been observed due to warming climate on one hand and their inherent stochastic nature on the other hand. Discharge changes affect many other climatic and environmental phenomena such as runoff, flood, and humidity and also affect many human activities such as agriculture, economics, soil erosion and etc [2]. Projecting future inflows under climate change effects and rapidly growing population has large uncertainty and requires serious attention on proper utilization of limited water resources [3].

Reservoir systems are usually characterized by multiple

Review History:

Received: 5 November 2018 Revised: 26 January 2019 Accepted: 27 January 2019 Available Online: 27 January 2019

Keywords: Many-Objective Uncertainty Scenarios Melody Searc Social Choice

objectives that often conflict and compete with one another [4]. The problems with a small number of objectives, mainly in two or three objectives are referred to as Multi-Objective Problems (MOP). However, many real-world applications often involve four or more objectives, which are commonly called as Many-objective Optimization Problems (MaOP) [5].

A review of the literature shows that multi-objective system operation has been extensively studied using various optimization algorithms. In the early stages of system optimization studies in water resources, weighting approach or constraint method were used with linear programming (LP) [6], dynamic programming (DP) [7, 8], stochastic dynamic programming (SDP) [9-11] and various nonlinear programming (NLP) techniques [12]. These methods convert the multi-objective problem into a single objective via various techniques. In these cases, the solution is highly dependent on weight vector and user knowledge. There was a turnover in the early 1990s when evolutionary methods were introduced. Thereafter in less than a decade, an explosion of research was directed toward the development and application of these methods. This turnover was mainly due to simple structure, least knowledge of mathematics required, flexibility, and adaptability inherent in heuristic methods. Heuristic algorithms are mainly inspired by the natural systems

Corresponding author, E-mail: fkarami@kntu.ac.ir

and have proved to be powerful tools for solving complex problems that were once considered very hard to solve. There have also been methods that were inspired by manmade systems and processes. Simulated Annealing (SA) [13] and Harmony Search [14](HS) are in this category.

Many-objective optimization has been gaining increasing attention in recent years. In many-objective optimization problems, the proportion of non-dominated objective solutions increases rapidly with the number of objectives [15–17]. This leads the Pareto optimality to significantly diminishing selection pressure during the evolutionary process [18-20]. Zhou et al. [18] used a combination of Pareto dominance selection, diversity maintenance, and elitism strategy and proposed an ensemble of manyobjective evolutionary algorithms (EMaOEA). The hypervolume (HV) indicator [21, 22] is a quality indicator that is fully sensitive to Pareto dominance. There have been several well-established hypervolume-based MOEAs available in the literature [23–27]. The main drawback of this indicator is the computational cost of HV which grows exponentially with the number of objectives increasing [28, 29]. To address this issue, Bader and Ziztler [30] proposed a fast search algorithm that uses Monte Carlo simulation to approximate the exact hypervolume values. The hypervolume indicator gives the volume of the objective subspace that is dominated by a solution R^d under consideration. Grid-based algorithms set A exploit the potential of the grid-based approach to strengthen the selection pressure towards the optimal direction while maintaining an extensive and uniform distribution among solutions. Each solution in the grid has a deterministic location [31]. The number of solutions whose grid locations are analogous reflects diversity. Also, the grid location of an individual compared with other solutions, determines the convergence. This approach compares solutions qualitatively and quantitatively [28, 29]. Deb and Jain [32] proposed NSGA-III on the basis of the NSGA-II algorithm with significant changes in its selection mechanism for many-objective problems. This algorithm uses a predefined set of reference points H on a unit hyper-plane to ensure diversity in the solutions. The reference points can be predefined in a structured manner or by the user [33]. Ruiz et al. [34] used the reference points and suggested a preference-based evolutionary multiobjective optimization called weighting achievement scalarizing function genetic algorithm.

The social choice theory is in close relation with multiple objective algorithms, especially in group decision contexts. The SC procedures are voting systems for group decisionmaking when available information is minimal, or mainly qualitative [35]. The subject is to derive social orderings when individual welfares satisfy certain assumptions [36].

Existing algorithms can optimize the operation policy only for a predetermined specified scenario (i.e., dry, wet or normal year etc) and do not work well in stressful conditions (when demand is high). This article aims at providing an optimization algorithm that solves this problem by using a combined model of social choice and melody search methods. The proposed algorithm optimizes reservoir operation policy considering various uncertainty scenarios. This leads to a readiness to deal with them in advance. As it was mentioned earlier, when the number

of objectives increases, the proportion of non-dominated objective solutions grows rapidly. Existing many-objective algorithms use special methods to overcome this issue which includes high computational costs. Meanwhile, it should be noted that the proposed algorithm has no restrictions on the number of functions. Moreover, on issues related to water resources and specifically reservoir management, objective functions have weighted priorities and the weight assignment of each consumer is important. Finding appropriate weights requires sensitivity analysis and multiple runs in traditional algorithms which includes high computational costs, but in the proposed method this is done by selecting the suitable social choice technique. Also, social choice approach is based on the consent of the individual components of the system. Easy perception, simple implementation, and rapid convergence are other advantages of this algorithm.

The paper is organized in the following way. Section 2 presents a brief discussion on methodology. The MeS and SC methods are presented in Sections 2.1 and 2.2. Section 2.3 presents a brief discussion on the proposed method considering various uncertainty scenarios. The case study is presented in Section 3 followed by results and discussion in Section 4. Finally, a conclusion is provided to summarize the important findings of the paper.

2- Methodology

2-1- Melody Search Algorithm (MeS)

Melody search algorithm was proposed by Ashrafi and Dariane (2013) on the basis of the harmony search algorithm [14]. However, the structure and efficiency of MeS are quite different from the harmony search algorithm. Harmony search is inspired by the improvisation process of jazz music and consists of the following steps [38–40].

- 1. Initialize the algorithm parameters. The parameters are the harmony memory size (HMS), harmony memory consideration rate (HMCR), pitch adjusting rate (PAR), distance bandwidth (BW), and termination criterion.
- 2. Initialize the harmony memory (HM).In Step 2, the "harmony memory" matrix is filled with randomly generated solution vectors and sorted in terms of the objective function value.
- 3. Improvise a new harmony from the HM. A new harmony is produced by applying three rules:memory consideration, pitch adjustment, and random selection. First of all, if a uniform random number returned by rand () in [0,1] is less than HMCR, the decision variable is generated by the memory consideration; otherwise, it is obtained by a random selection. Secondly, each decision variable will undergo a pitch adjustment with a probability of PAR if it is updated by the memory consideration.
- 4. Update the HM. If the new harmony vector is better than the worst harmony in the HM by the values of the objective function, the new harmony is included in the HM and the existing worst harmony is excluded from the HM then the harmony memory is sorted again.
- 5. Repeat Steps 3 and 4 until the termination criterion is satisfied.

MeS simulates musical performance processes performed in a musicians group while they are searching for the best series of notes within a melodic line. In such groups, each musician could be influenced by the others, and some interactive relations occur among different music players. In the MeS algorithm, particular emphasis is on simulation of these interactive relations through proper equations.

Unlike HS that uses a single Harmony Memory (HM), MeS algorithm employs several Player Memory (PM) sets. Existing different solutions in different memories would lead the search scheme to select more useful random variables if a suitable logical interactive relation is defined among various memories. Moreover, considering several memories with different historical experiences could increase the efficiency of the algorithm.

There are seven major parameters applied in MeS algorithm, including number of player memories (PMN), player memory size (PMS), maximum number of iterations (NII), maximum number of iterations for the initial phase (NI), bandwidth distance (bw), player memory considering rate (PMCR) and pitch adjusting rate (PAR). Main steps of MeS algorithm can be summarized as follows:

- nitializing the optimization problem and adopting algorithm parameters
- Phase 1;
 - 1.1. Initialize PMs with random solutions

1.2. Generate a new solution from each PM with specified improvisation operators

1.3. Update PMs

1.4. Repeat sub-steps 1.2 and 1.3 until the criterion for stopping the initial phase is satisfied (e.g. maximum number of iterations for initial phase (NII))

• Phase 2;

2.1. Determine the possible ranges of variables for randomization operator

2.2. Generate a new solution from each PM according to the calculated possible variable ranges

2.3. Update PMs

2.4. Repeat sub-steps 2.1 to 2.3 until the stopping criterion is satisfied (e.g. maximum number of iterations (NI))

In the initial phase, each player improvises his/her melody individually. In this phase, players do not influence each other. In the next phase, the new possible range for each variable is calculated from the best melody of each Player Memory PM. These ranges are changed through different iterations [37]. This process is shown in Figure 1 where D is the number of decision variables of the optimization problem.



Figure 1. Calculating possible ranges of variables (sub-step 2.1)

2-2-Social Choice

The beginnings of social choice theory can be traced back to the French Revolution when a French mathematician and political scientist, Jean-Charles de Borda, devised the Borda method in 1770 [41]. But it was about two centuries later where Arrow (1951) resurrect the use of social choice. The social choice (SC) procedures are voting systems for group decision-making when available information is minimal, or mainly qualitative [35]. The subject is to derive social orderings when individual welfares satisfy certain assumptions [36]. Several approaches have been proposed for SC such as plurality voting, the Hare system, the Borda count, pairwise comparisons voting and approval voting. Using different methods usually gives different results, hence, selecting the right method is very important [43]. All methods try to find the best alternative in accordance with the preferences of all individuals [44].

There is various exploratory work on voting schemes [45-47]. Kant and Lee (2004) demonstrated the relevance of a social choice approach to sustainable forest management. Goetz et al. (2008) presented the concept of sequential allocation rules developed in social choice theory. D'Angelo et al. (1998) applied five of the most popular methods of social choice to select the best alternatives in a forestry management problem. Srdjevic (2007) Linked analytic hierarchy process and social choice methods to support group decision-making in water management.

Brief overviews of the most prominent methods of social choice are described in the following paragraphs by considering an example involving elections which is explained by Malkevitch (2002) and illustrated in Figure 2a. The election problem has 5 candidates, 6 ballots and 55 voters.



Figure 2. Social choice methods

2-2-1-Plurality Voting

The plurality voting method selects candidate who gets the largest number of first-place votes (but not necessarily the majority of the first-place votes). By this method which simply uses information from the first row, as it can be seen from Figure 2a, candidate A (with 18 votes) is the winner. This concept can be formulated mathematically as follows where i and j are the candidates and ballots respectively. If i is the first priority of jth ballots then $a_{ij}=1$; if i is the second one then $a_{ii}=2$ and so on.

$$f(a_{ii})=1$$
 if $a_{ii}=1,0$ Otherwise (1)

$$P_{i} = \sum_{j=1}^{55} f(a_{ij})$$
 (2)

$$\mathbf{P}_{i}^{*} = \max_{i} \mathbf{P}_{i} \tag{3}$$

2-2-2-Approval Voting

This method is non-preferential because does not use all information directly but it is easy to understand and use. Each voter can approve of any number of candidates. The winner is the most-approved candidate. In this case, if voters approve only the first two rows in the table, then the ballots will be as Figure 2b. D is the winner with 27 votes.

2-2-3-Run-off Election

If no candidate receives the majority number of votes, then those candidates having less than a certain proportion of the votes, or all but the two candidates receiving the most votes, are eliminated, and a second round of voting is held. In this example, A (18 votes) and B (12 votes) are selected first and then in the next stage, B becomes the winner with 37 votes. It is worthwhile to mention that after eliminating other candidates, A only receives votes from the first ballot (i.e., 18 votes), while B becomes the first priority in all the remaining 5 ballots and receives 37 votes (i.e., 12+10+9+4+2). This process is illustrated in Figure 2c.

2-2-4-Sequential Run-off Election

In this method, if no candidate gets a majority based on first-place votes, candidates are eliminated one by one. First, the candidate with the fewest first-place votes is eliminated and a new election based on voting only for the remaining collection of candidates is held. In this case, first candidate E (6 votes) and then D (9 votes) and B (16 votes) are eliminated respectively. Eventually, C is the winner with 37 votes as shown in Figure 2d.

2-2-5-Condorcet (Pairwise voting)

All possible two-way races between candidates are considered and the candidate that would win in all pairings against the other candidates is called non-dominated and elected. It should be noted that in each race between two candidates all other candidates are eliminated and the votes in each type are given to the first priority candidate. In this case, there are 20 combinations of two-way races and E is the final winner. Since the majority preferences can be like rock-paper-scissors a Condorcet winner doesn't always exist (Condorcet paradox).

2-2-6-Borda Count

This method assigns candidate "i" a number of points equal to the number of candidates below candidate "i" on the ballots. The winner of the election is the candidate with the highest Borda count. For example, in this case, the Borda count of D is 136 ($3\times18+2\times12+1\times10+4\times9+2\times4+2\times2$) and is higher than A(72), B(101), C(107) and E(134). Hence, D is the winner (Figure 2e).

Borda method is the only method that simultaneously uses all data in the priorities table. It is proved that Borda

count and approval voting methods are more effective in determining the winner than the other methods [51]. Moreover, in the Borda count method, priorities could be weighted.

2-2-7-Proposed Method

Existed algorithms can optimize the operation policy only for a predetermined specified scenario (i.e., dry, wet or normal year) and do not work well in stressful conditions (when demand is high). Therefore, when the basin condition changes they are unable to adapt to the situation and fail to operate properly. Moreover, some of the water resources systems involve more than three objectives that demand the use of many-objective algorithms. The proposed method overcomes these issues by innovative use of together melody search algorithm and social choice methods to solve a many-objective reservoir operation problem under various probable scenarios. Thus, the derived rules would work properly and optimal under different basin conditions.

The proposed algorithm consists of the following steps as shown in Figure 4:

- Initializing the optimization problem and determining algorithm parameters.
- Generating various scenarios (by systematically varying inflows and demands.)
- Phase 1 (each player improvises his/her melody individually.);
 - 1.1. Initialize PMs with random solutions.

1.2. Check constraints and adjust solutions if necessary.1.3. Calculate all objectives functions for every existing solution and different scenarios.

1.4. Rank player memories based on different objective functions for each scenario individually. For example for "m" objective functions, solutions should be sorted m times for each scenario.

1.5. Combine the results of different objective functions based on SC methods for each PM and each scenario separately.

Use the Borda count method for sorting solutions in each scenario based on objective functions. If the Borda count of two solutions is the same, the winner is selected by the approval voting method based on objective function plurality. Hence, if m and s are the number of objectives and scenarios respectively, we have s sorted memory for each PM. It should be noted that in this way the proposed algorithm has no restrictions on the number of objective functions.

Other methods of SC can be used instead of the Borda count method. Since the objective functions have priorities in water resources systems and specifically reservoir management problems, selecting the suitable social choice technique is an alternative for sensitivity analysis for finding appropriate weights of each consumer. Finding optimum weights requires multiple runs in traditional algorithms that demand high computational costs.

1.6. Combine the results of different scenarios based on Borda counts for each PM. Each solution has different ranks in harmony memory based on different scenarios. Borda count is calculated based on these ranks for each solution in each PM. Then PM memory is sorted based on these Borda counts where solutions with higher Borda counts are preferred. Figure 3 shows steps 1.4 to 1.6.

1.7. Generate a new solution from each PM with specified improvisation operators.

1.8. The Borda count of the new solution is calculated for each scenario based on different objective functions. If the Borda count of the new solution is better than the worst one in the memory for each scenario, that scenario preliminary votes to replace the new solution with the existing worst solution. However, the final decision depends on the collective vote of all scenarios. Thus, based on votes of all scenarios (plurality voting method) the decision for replacing the new solution with the existing worst solution is made.

1.9. Repeat sub-steps 1.3 to 1.7 until the criterion for stopping the initial phase is satisfied (e.g., the maximum number of iterations for the initial phase (NII)).

Phase 2 (a suitable logical interactive relation is defined among various memories);

2.1. Determine the possible ranges of variables for randomization operator.

2.2. Generate a new solution from each PM according to the possible variable ranges as defined in the previous step.

2.3. Decide if the new solution would replace the existing worst solution or not, based on scenarios votes, similar to phase 1.

2.4. Sort memory by using social choice techniques. In this phase unlike the initial phase that the solutions were sorted by Borda count, the plurality voting method is employed and accordingly, the solution with higher total rank of the first and second objective functions is preferred over the other one. In this way, the priority of functions is considered.

2.5. Combine the results of different scenarios based on Borda counts.

2.6. Repeat sub-steps 2.1 to 2.5 until the stopping criterion is satisfied (e.g., maximum number of iterations (NI))

Figure 4 shows the schematic flowchart of the algorithm.



Figure 3. Individually ranking player memories (PM) based on different objective functions for each scenario and then combining the results of different scenarios based on Borda counts for each PM

3- Case Study

To evaluate the performance of the proposed algorithm, a five-reservoir system including Taleghan, Karaj, Lar, Latian and Mamloo reservoirs within the Tehran region is employed as the case study. The area is located in northern Iran at 35°45′ northern latitude and 51°30′ eastern longitude. Figure 5 schematically shows the location map of the system under study.

The region has municipal, agricultural and hydropower demands. Among these reservoirs, Lar suffers from excessive water escape and is unable to effectively store the water. In this site, water is transferred to Tehran using a channel with maximum capacity and the remaining is transferred to the Latian reservoir, again up to its maximum channel capacity. Moreover, Taleghan and Karaj reservoirs act in parallel in meeting the municipal demand of Tehran. The water is transferred from these reservoirs by channels with limited capacities that must be considered in modeling the system. Similarly, Latian and Mamloo act as parallel in meeting Tehran water demand, although they are in cascade on Jajrud River. There are three agricultural sites in the region namely; Taleghan, Karaj and Varamin areas. Meanwhile, maintaining a minimum instream flow for environmental purposes is also required. In addition, three hydropower plants on Karaj, Taleghan and Lar reservoirs generate part of the electricity demand of the system. Municipal demands of Tehran have been estimated from the previous studies and are shown in Table 1 along with long-term monthly average inflows at different sites.



Figure 4. Schematic flowchart of algorithm



Figure 5. Tehran location map

Table 5. Soil properties in numerical modeling

	Inflows to th	ne reservoirs a (S	nd maximum max and Smi	Demands					
month	Taleghan	leghan Karaj		Latian Mamloo					
	Smax=420	Smax= 205	Smax=95	Smax=250	Smax= 960	Tehran	Taleghan	Karaj	Varamin
	Smin=100	Smin=10	Smin=20	Smin=28	Smin=10	Municipal	ag.	ag.	ag.
1	10.4	13	9.4	3.8	14.6	85.3	10	3	5.3
2	14	14.6	13.3	5.5	13	79.9	3.5	1	6.3
3	12.7	13.5	13	5.7	11.1	77.8	0	0	3.4
4	11.5	12.3	12.7	5.2	9.6	77.5	2.8	0.8	3.7
5	12.9	13	15	6.4	8.8	78	2.9	0.9	7.9
6	24	22.8	30	12.8	10.4	81.3	7.1	2.1	15.8
7	78.7	57.5	72.8	28.6	39.9	76.8	27.7	8.3	40.4
8	138.6	92.5	84.3	33.8	110.3	84.2	41.4	12.3	56.6
9	99.2	78.4	48.8	18.7	105.5	92.6	54.4	16.2	51.9
10	39.1	45.4	21	7.2	50.6	100	65.8	19.6	50.6
11	16.7	24.4	11.6	4	28.9	101	61.6	18.4	34.9
12	10.4	16.3	9.1	3.1	20.5	96.2	32.8	9.8	17.8
annual	468.2	403.8	341.2	134.8	423.3	1030	310	92.4	294.6

The objective function of the system is reliability. Reliability is the probability that the reservoir provides the outflow required to satisfy the water demand. This parameter is obtained by dividing the number of successful periods into the total number of periods. Minimum flow requirements are determined for aquatic resources based on 10 percent of mean annual flow. In this system, the first priorities are given to satisfy the municipal demand of Tehran city with maximum reliability throughout the system and environmental minimum instream flow requirements. The next priority is to fulfill the agricultural demands, and finally, the hydropower energy production receives the least priority. The constraints are channel capacities, mass balance, and storage capacities.

Release policies are usually derived using predefined rule curves based on the system states as defined by reservoir storages and inflows. In this study, a linear operating rule is defined for each reservoir release as follows.

$$R_{(y,m)} = a_{(m)} * S_{(y,m)} + b_{(m)} * Q_{(y,m)} + c_{(m)} * Q_{(y,m-1)} + d_{(m)}$$
(4)

Where y and m denotes year and month indexes respectively, and a, b, c, and d are the policy parameters determined by the optimization model. $S_{(y,m)}$ is the storage level at the beginning of period m in year y. $R_{(y,m)}$ and $Q_{(y,m)}$ are the total release and the inflow during period m, respectively.

4- Results and discussions

In this section, the performance of the proposed procedure which employs combined use of MeS and SC methods is evaluated by comparing to other alternative methods. For this purpose 35 years of available monthly data are used for training and calibration, and the remaining 12 years of data are used for testing and cross-validation of the results of trained models. Moreover, the number of player memories (PMN), player memory size (PMS), maximum number of iterations (NII), maximum number of iterations for the initial phase (NI), bandwidth distance (bw), player memory considering rate (PMCR) and pitch adjusting rate (PAR) program are obtained by sensitivity analysis as 5, 10, 8000, 30000, 0.1, 0.98 and 0.3 respectively. All the results shown in tables are for the test period and are obtained by 10 independent model executions.

In order to evaluate the performance of the proposed model in finding optimum reservoir operating policies for Tehran water resources system with multiple reservoirs and multiple purposes, two steps are followed. In the first step, the performance of the proposed algorithm is evaluated and compared to other existed multiple objective methods. Moreover, the performance of the melody search algorithm as part of the proposed model is compared with other heuristic methods including genetic algorithm (GA) and harmony search in combination with the social choice and weighting methods (WM). In the next step, the capability of handling multiple scenarios by the proposed model is demonstrated. Here, the reservoir's operation policies are derived using various uncertainty scenarios combined. For this purpose, seven "hydroclimatic scenarios" are defined based on decreasing inflows to resemble the climate change impacts and another seven "population growth scenarios" are specified based on increasing municipal and agricultural demands. Results are orderly presented in the following sections.

4-1-Comparison of Algorithms

In this section, the performance of the proposed model is compared to the commonly applied multi-objective Nondominated Sorting Genetic Algorithm-II (NSGA-II). The non-dominated sorting genetic algorithm (NSGA) proposed by Srinivas and Deb [52], is one of the first evolutionary-based multi-objective algorithms. Deb et al. [53] proposed NSGA-II as an improved version of NSGA. For this purpose, first, the system reservoir operation policy was optimized considering a two-objective function problem consisting of the reliabilities for municipal and hydropower. The minimum point in the average Euclidean distance vector between solutions in Pareto and the ideal point was picked for comparison. The ideal point consists of the best value of each objective functions. It should be noted that parameters are obtained by sensitivity analysis and the number of function evaluations in both algorithms is the same.

As can be seen from Table 2, the proposed model and the NSGA-II show similar performances when there are only two objective functions. However, when the number

of objectives is increased to four (including municipal. minimum instream flow, agricultural and hydropower energy) and the problem becomes more complicated, the NSGA-II fails to reach solution easily obtained by the proposed model (see Table 2). This indicates the distinguished power of the proposed algorithm in the modeling of many-objectives problems. As it was mentioned earlier, when the number of objectives increases, the proportion of non-dominated objective solutions increases rapidly in Pareto based algorithms including the NSGA-II. This leads the algorithm to significantly diminishing selection pressure during the evolutionary process. Selection pressure reduction potentially reduces reproductive success in a proportion of the population. This is an advantage for the proposed model which does not require a Pareto front for solving a many-objective problem.

It should be noted that parameters for both algorithms are obtained by sensitivity analysis first for two objective functions and then for four ones.

The performance of investigated methods including GA, HS, and MeS in combination with SC and WM is compared next. The objective functions are assumed to be the maximization of long-term system reliabilities for municipal, minimum instream flow, agricultural and hydropower energy demands. As was mentioned earlier, in the weighting method, multiple objective functions are combined into one overall function by using weighted aggregation of the objective functions depending on their importance [54]. It should be noted that for the weighting method, a sensitivity analysis is needed to find the optimum weights (0.4, 0.3, 0.2 and 0.1 for municipal, environmental, agricultural and hydropower demands respectively). Table 3 shows the results.

As can be seen from Table 3, using social choice improves results for all demands. The difference is more evident for municipal demand which has the first priority. Moreover, MeS shows generally better performance than either of other heuristic algorithms including HS or GA algorithms in this study.

Finally in SC algorithms, finding appropriate weights is replaced by selecting the suitable social choice technique but WM methods require sensitivity analysis and multiple runs. In the proposed algorithm, Borda count is used for the iterations in phase 1 and for phase 2 iterations, plurality voting method is employed. Here, the first two objectives having higher priority are selected for phase 2 analysis. Then, accordingly, the solutions with higher total rank based on these objective functions are preferred over the other ones. Actually, the SC algorithms can draw the consent of the individual components of the system and reach the solution in much less time.

	Two Objective Functions			Four Objective Functions			
Algorithm	Municipal	Hydropower	Municipal	Minimum flow	Agricultural	Hydropower	
NSGAII	0.82	0.78	0.64	0.56	0.42	0.79	
MeS, SC	0.82	0.79	0.72	0.59	0.4	0.79	

Table 2. Comparing the performance of the algorithm under multi and many-objective problems

Mathad	Reliability							
Method	Municipal	Minimum flow	Agricultural	Hydropower				
GA, WM	0.57	0.48	0.35	0.73				
GA, SC	0.66	0.51	0.38	0.79				
HS, WM	0.60	0.43	0.34	0.73				
HS, SC	0.66	0.51	0.40	0.78				
MeS, WM	0.64	0.54	0.37	0.74				
MeS, SC	0.72	0.59	0.40	0.79				

Table 3. Comparing the performance of different algorithms for Tehran system

4-2-Uncertainty Scenarios

In this section, the proposed algorithm optimizes reservoir operation policy considering various uncertainty scenarios. This leads to a readiness to deal with them in case of occurrence. Meanwhile, existed algorithms can optimize the operation policy only for a specified scenario. They can consider only one possible scenario for deriving the system operation rule. Thus, they fail to function if the system condition changes. In other words, the rules derived say for a normal condition would fail to properly work for drought situation and vice versa. For this purpose, seven "hydroclimatic scenarios" are defined based on decreasing inflows to resemble climate change impacts and seven "population growth scenarios" are defined based on increasing municipal and agricultural demands. The current condition is also assumed to form the first scenario which is common between the two scenario categories. To clear up the current condition is the first scenario of hydroclimatic scenario. The second one is prepared by decreasing 20 percent of inflow. The third one is prepared by decreasing 40 percent of inflow and etc. in population growth scenarios the first scenario is the current condition. The second one is prepared by increasing 20 percent of demands. The third one is prepared by a 40 percent increase in demands and etc.

In order to demonstrate the power of the proposed algorithm, the decision parameters and thus the operation rules that were optimized by considering all scenarios, are tested under each possible scenario separately. Moreover, for comparison purposes, an operation rule is also obtained by using the third version of the Non-dominated Sorting Genetic Algorithm (NSGA-III), as a recently developed powerful many-objective algorithms, for every single scenario. The operation rule derived by each scenario through this process is also tested assuming the occurrence of any other scenario. It should be noted that NSGA-III similar to all other existing algorithms can optimize the operation policy only for a single predetermined climate scenario. The minimum point in the average Euclidean distance vector between solutions in Pareto and the ideal point was picked for comparison. The ideal point consists of the best value of each objective functions. Results are presented in Table 4 for hydroclimatic scenarios. The percent of variation between each objective function and the best value found under a special scenario are shown in the Tables using color trends. It is interesting to note that by applying the proposed algorithm (model 1) where the decision variables are derived using all possible scenarios, the objective functions show the best

performance (highest reliability) for all scenarios. But other models which employ a specified scenario to derive the rules, just work well under that scenario. For example, using the proposed algorithm, the municipal reliability (the first objective function) is 0.72 under scenario 1, 0.65 under scenario 2 and so on. It is obvious from this Table that the reliability values obtained by the proposed method for each hydroclimatic scenario is maximum among all other models. In other words, the maximum value in each scenario column occurs in the proposed model row. However, other models also reach to nearly the same reliability value that they are based on. In all other cases where the scenario changes to other than the one employed by the model, the model performance is very poor as compared to the proposed model. For example, model 2 which is based on scenario 1 (current condition) shows a reliability value of 0.72 (equal to the value obtained by the proposed method) if that scenario occurred, but in all other cases where a different scenario takes place, the reliability values are well below those of the proposed method. The same is true for model 3 which is based on scenario 2 where the best performance also occurs if scenario 2 takes place. In this case, a reliability value of 0.67 nearly the same as that obtained by the proposed method (i.e., 0.65) is obtained but in all other scenarios, the performance of model 3 is poor. Similar results are observable in all other models for municipal reliability. Also, the same trend is evident for other water demand sectors including the environmental (minimum flow), agricultural and hydropower sectors. Nevertheless, It is clear that when the operation rule is optimized under one scenario, optimal value should be obtained in that scenario for simulation phase but as one can see from the Table, the proposed algorithm can perform well under all scenarios.

In Table 4, color trends indicate that as we move further from the scenario that was used to derive the system operation rules, the results are further deviated from the best performance in each case (column). Interestingly, the rules derived for normal or near-normal hydroclimatic conditions show deteriorations when facing dryer conditions. This is more observable in case of municipal demands where higher demands are requested with the first priority than other sectors either with less amount of demand and/or lower priority. Yet, the hydropower sector experience similar trends as the municipal sector but to a lesser extent, since part of the water released for municipal use also is used for hydropower generation. Nevertheless, it is also observable from the results that in general the rules derived for dryer conditions work relatively better in wet conditions than vice versa. This could be due to the hedging effect of dry condition rules that try to save more water in the reservoir storages for future conditions.

Similar trend is observed in population growth scenarios as shown in Table 5. As can be seen the percent of variation in Table 5 changes in a wider range than Table 4. According to these results, the system is more sensitive to demands changes than the inflow changes for the municipal objective. Moreover, rules derived for normal population conditions (i.e., first three scenarios) fail to function properly in higher population conditions (i.e., last three scenarios); similar to drought and wet conditions in the hydroclimatic scenarios as mentioned earlier in Table 4.

Table 4. Results of objective functions for hydroclimatic scenarios									
	Ontimization Simulation with each hydroclimatic scenario						enario		
	model	based scenario	scenario 1	scenario 2	scenario 3	scenario 4	scenario 5	scenario 6	scenario 7
	1	proposed method (all scenarios)	0.72	0.65	0.59	0.48	0.4	0.3	0.21
-	2	scenario 1	0.72	0.4	0.32	0.23	0.19	0.15	0.1
-	3	scenario 2	0.69	0.67	0.36	0.24	0.22	0.15	0.1
Municipal -	4	scenario 3	0.68	0.4	0.59	0.32	0.23	0.15	0.1
-	5	scenario 4	0.6	0.38	0.37	0.49	0.24	0.19	0.12
-	6	scenario 5	0.51	0.37	0.32	0.29	0.39	0.2	0.14
-	7	scenario 6	0.49	0.35	0.3	0.28	0.28	0.29	0.14
-	8	scenario 7	0.47	0.32	0.29	0.28	0.27	0.2	0.23
	1	proposed method (all scenarios)	0.59	0.54	0.5	0.44	0.37	0.3	0.2
-	2	scenario 1	0.59	0.47	0.42	0.37	0.3	0.24	0.16
Indraam	3	scenario 2	0.57	0.52	0.44	0.39	0.31	0.24	0.16
Requirement	4	scenario 3	0.59	0.47	0.47	0.38	0.31	0.24	0.16
Reliability	5	scenario 4	0.55	0.47	0.44	0.42	0.33	0.25	0.18
-	6	scenario 5	0.5	0.47	0.42	0.36	0.36	0.25	0.18
-	7	scenario 6	0.53	0.46	0.42	0.38	0.33	0.29	0.18
	8	scenario 7	0.53	0.43	0.4	0.38	0.33	0.25	0.19
	1	proposed method (all scenarios)	0.4	0.31	0.29	0.23	0.2	0.15	0.12
-	2	scenario 1	0.39	0.3	0.29	0.2	0.18	0.12	0.09
-	3	scenario 2	0.36	0.31	0.28	0.2	0.18	0.13	0.1
Agricultural	4	scenario 3	0.36	0.3	0.29	0.24	0.18	0.13	0.1
Kenability -	5	scenario 4	0.35	0.28	0.27	0.24	0.19	0.13	0.1
-	6	scenario 5	0.33	0.27	0.26	0.23	0.2	0.14	0.11
-	7	scenario 6	0.33	0.26	0.25	0.2	0.17	0.15	0.11
-	8	scenario 7	0.33	0.25	0.24	0.2	0.15	0.14	0.11
	1	proposed method (all scenarios)	0.74	0.7	0.65	0.64	0.56	0.48	0.42
	2	scenario 1	0.76	0.68	0.62	0.54	0.46	0.33	0.22
	3	scenario 2	0.72	0.72	0.62	0.57	0.47	0.37	0.25
Hydropower	4	scenario 3	0.7	0.67	0.63	0.61	0.48	0.37	0.25
Reliability	5	scenario 4	0.66	0.67	0.63	0.61	0.54	0.47	0.33
	6	scenario 5	0.66	0.67	0.63	0.61	0.57	0.48	0.33
-	7	scenario 6	0.54	0.55	0.54	0.51	0.51	0.49	0.37
-	8	scenario 7	0.47	0.48	0.46	0.46	0.5	0.48	0.44
Legend		Variation percent		<10%	10-20%	20-30%	30-40%	40-50%	>50%
Legena		Color							

	Table 5. Results of objective functions for population growth scenarios									
	Simulation with each popula					ch populat	tion growth scenario			
	model	based scenario	scenario 1	scenario 2	scenario 3	scenario 4	scenario 5	scenario 6	scenario 7	
	1	proposed method (all scenarios)	0.72	0.56	0.48	0.34	0.23	0.17	0.13	
-	2	scenario 1	0.72	0.45	0.19	0.12	0.03	0.05	0.02	
-	3	scenario 2	0.67	0.55	0.23	0.15	0.1	0.02	0.01	
Municipal	4	scenario 3	0.67	0.37	0.47	0.18	0.16	0.03	0.03	
Reliability	5	scenario 4	0.63	0.31	0.26	0.35	0.16	0.1	0.07	
-	6	scenario 5	0.63	0.3	0.26	0.19	0.25	0.11	0.08	
-	7	scenario 6	0.63	0.27	0.26	0.19	0.17	0.18	0.12	
	8	scenario 7	0.57	0.23	0.22	0.18	0.17	0.15	0.13	
	1	proposed method (all scenarios)	0.59	0.55	0.53	0.51	0.49	0.49	0.49	
-	2	scenario 1	0.63	0.5	0.49	0.48	0.47	0.43	0.44	
Instroom	3	scenario 2	0.63	0.55	0.46	0.45	0.45	0.43	0.44	
Requirement	4	scenario 3	0.63	0.55	0.54	0.5	0.51	0.44	0.45	
Reliability	5	scenario 4	0.6	0.54	0.52	0.51	0.5	0.44	0.44	
-	6	scenario 5	0.59	0.51	0.5	0.49	0.49	0.45	0.45	
	7	scenario 6	0.57	0.49	0.47	0.46	0.45	0.47	0.45	
	8	scenario 7	0.53	0.46	0.45	0.44	0.43	0.43	0.46	
	1	proposed method (all scenarios)	0.42	0.39	0.35	0.34	0.33	0.3	0.3	
	2	scenario 1	0.73	0.37	0.33	0.33	0.3	0.3	0.29	
	3	scenario 2	0.42	0.4	0.34	0.32	0.32	0.31	0.28	
Agricultural	4	scenario 3	0.4	0.41	0.36	0.33	0.33	0.29	0.26	
Kenability	5	scenario 4	0.4	0.4	0.35	0.34	0.33	0.3	0.28	
_	6	scenario 5	0.39	0.36	0.32	0.33	0.33	0.3	0.28	
_	7	scenario 6	0.38	0.34	0.32	0.31	0.28	0.3	0.29	
	8	scenario 7	0.39	0.33	0.35	0.29	0.28	0.31	0.3	
	1	proposed method (all scenarios)	0.71	0.69	0.67	0.63	0.62	0.6	0.58	
	2	scenario 1	0.72	0.69	0.67	0.63	0.57	0.53	0.5	
	3	scenario 2	0.68	0.71	0.66	0.63	0.62	0.53	0.5	
Hydropower Reliability	4	scenario 3	0.68	0.69	0.69	0.63	0.6	0.56	0.53	
Reliability	5	scenario 4	0.68	0.69	0.69	0.65	0.62	0.58	0.53	
	6	scenario 5	0.65	0.69	0.68	0.64	0.63	0.62	0.58	
-	7	scenario 6	0.63	0.69	0.65	0.63	0.63	0.62	0.59	
	8	scenario 7	0.62	0.69	0.63	0.6	0.62	0.61	0.59	
		Variation percent		<20%	20-30%	30-40%	40-50%	50-70%	>70%	
Legend		Color								

In this paper, a novel many-objective optimization algorithm was introduced that is able to derive the operation policy of systems with multiple reservoir considering various future scenarios. The model was

5- Conclusion

evaluated using a five-reservoir system in the Tehran region with several objectives including meeting municipal, agricultural, environmental, and hydropower demands. For this purpose, two groups of scenarios including seven "hydroclimatic scenarios" and another seven "population growth scenarios" were defined, based on decreasing inflows to resemble climate change impacts and increasing municipal and agricultural demands, respectively. Then, the performance of various methods including GA, HS, and MeS in combination with SC and WM was evaluated. Results indicated that using social choice coupled with heuristic algorithms improves the system performance especially for the municipal demand which has the first priority. Moreover, the MeS algorithm showed generally better performance than either of other heuristic algorithms including HS and GA. Finally, it was shown that in all cases the multi-scenario rule derived by the proposed method performs as good as the operation rule derived for any specific scenario when evaluated for that scenario. Finally, a comparison of results revealed the superiority of the proposed model over NSGA-II as the number of objectives increased from two to four (i.e., becoming more complicated). For instance, the proposed model was able to reach a reliability level of 0.72 in contrast to 0.64 obtained by the NSGA-II in meeting the municipal demand of Tehran city. The proposed method is more appreciated when the system planning (sizing the hydro structures) is considered.

References

- [1]B. Wang, W. Li, G.H. Huang, L. Liu, L. Ji, Y. Li, Urban water resources allocation under the uncertainties of water supply and demand: a case study of Urumqi China, Environmental Earth Sciences, 74 (2015) 3543-3557.
- [2] M. Nazeri Tahroudi, F. Ahmadi, K. Khalili, Impact of 30 Years Changing of River Flow on Urmia Lake Basin, AUT Journal of Civil Engineering, 2 (2018) 115-122.
- [3] A. Nazemi, H.S. Wheater, How can the uncertainty in the natural inflow regime propagate into the assessment systems?, Advances of Water Resources, 63 (2014) 131-142.
- [4]L.C. Chang, F.J. Chang, Multi-objective evolutionary algorithm for operating parallel reservoir system, Journal of Hydrology, 377 (2009) 12–20.
- [5]C. Dai, Y. Wang, L. Hu, An improved α-dominance strategy for many-objective optimization problems, Soft Computing, 20 (2016)1105–1111.
- [6]K.W. Wang, L.C. Chang, F.J. Chang, Multi-tier interactive genetic algorithms for the optimization of long-term reservoir operation, Advances of Water Resources, 34 (2011) 1343–1351.
- [7]H. Daellenbach, A. Kluyver, Note on multiple objective dynamic programming, Journal of the Operational Research Society, 31 (1980) 591–594.
- [8] B. Villarreal, M. Karwan, Multicriteria integer programming: a hybrid dynamic programming recursive approach, Mathematical Programming, 21 (1981) 204–223.
- [9] B. Talkudar, D. Deb, S. DK, Development of multiobjective stochastic dynamic programming (MOSDP) reservoir operation model, in World Environmental and Water Resources Congress, (2012) 985-997.
- [10] A. Serrat-Capdevila, J.B. Valdes, An alternative approach to the operation of multinational reservoir systems: Application to the Amistad and Falcon system (Lower Rio Grande/Rio Bravo), Water Resources Management, 21 (2007) 677–698.

- [11] N. Saadouli, C. Edirisinghe, Multi-stage stochastic model for a multipurpose water reservoir with target-priority operation, Water, (2006) 146–151.
- [12] J.W. Labadie, Optimal Operation of Multireservoir Systems: State-of-the-Art Review, Journal of Water Resources Planning Management, 130 (2004) 93–111.
- [13] P.J van Laarhoven, E.H. Aarts, Simulated Annealing: Theory and Applications, Springer Netherlands, (1987) 7-15.
- [14] Z.W. Geem, J.H. Kim, G.V. Loganathan, A New Heuristic Optimization Algorithm: Harmony Search, Simulation, 76 (2001) 60–68.
- [15] P.J. Fleming, R.C. Purshouse, R.J. Lygoe, M. Street, S. Sheffield, Many-objective optimization: an engineering design perspective, Lecture Notes in Computer Science, 3410 (2005) 14–32.
- [16]S.F. Adra, P.J. Fleming, Diversity management in evolutionary many-objective optimization, IEEE Transactions on Evolutionary Computation, 15 (2011) 183–195.
- [17]Z. He, G.G. Yen, J. Zhang, Fuzzy-based pareto optimality for many-objective evolutionary algorithms, IEEE Transactions on Evolutionary Computation, 18 (2014) 269–285.
- [18] Y. Zhou, J. Wang, J. Chen, S. Gao, L. Teng, Ensemble of many-objective evolutionary algorithms for many-objective problems, Soft Computing, 21(9) (2015) 2407-2419.
- [19] M. Li, S. Yang, X. Liu, Shift-based density estimation for pareto-based algorithms in many-objective optimization, IEEE Transactions on Evolutionary Computation, 18 (2014) 348–365.
- [20] X. Ma, Y. Qi, L. Li, F. Liu, L. Jiao, J. Wu, MOEA/D with uniform decomposition measurement for many-objective problems, Soft Computing, 18 (2014) 2541–2564.
- [21] E. Zitzler, L. Thiele, Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach, IEEE Transactions on Evolutionary Computation, 3 (1999) 257–271.
- [22] E. Zitzler, L. Thiele, Multi-objective optimization using evolutionary algorithms- a comparative case study, Parallel Problem Solving from Nature, Springer, Berlin, Heidelberg, (1998) 292–301.
- [23] N. Beume, B. Naujoks, M. Emmerich, SMS-EMOA: Multiobjective selection based on dominated hypervolume, European Journal of Operational Research, 181 (2007) 1653–1669.
- [24] C. Igel, N. Hansen, S. Roth, Covariance matrix adaptation for multi-objective optimization, Evolutionary Computation, 15 (2007) 1–28.
- [25] E. Zitzler, S. Künzli, Indicator-based selection in multiobjective search, Parallel Problem Solving from Nature, Springer, Berlin, Heidelberg, (2004) 832–842.
- [26] M. Emmerich, N. Beume, B. Naujoks, An EMO algorithm using the hypervolume measure as selection criterion, Evolutionary Multi-Criterion Optimization, (2000) 32–76.
- [27] D. Brockhoff, E. Zitzler, Improving hypervolume-based multiobjective evolutionary algorithms by using objective reduction methods, Congress on Evolutionary Computation (CEC), (2007), 2086–2093.

- [28] Y. Yuan, H. Xu, B. Wang, B. Zhang, X. Yao, Balancing convergence and diversity in decomposition-based manyobjective optimizers, IEEE Transactions on Evolutionary Computation, 20 (2016) 180–198.
- [29]Y. Yuan, H. Xu, B. Wang, X. Yao, A new dominance relation-based evolutionary algorithm for many-objective optimization, IEEE Transactions on Evolutionary Computation, 20 (2016) 16–37.
- [30] J. Bader, E. Zitzler, HypE : An algorithm for fast optimization, Evolutionary Computation, 19 (2011) 45–76.
- [31]S. Yang, M. Li, X. Liu, J. Zheng, A grid-based evolutionary algorithm for many-objective optimization, IEEE Transactions on Evolutionary Computation, 17 (2013) 721–736.
- [32]K. Deb, H. Jain, An evolutionary many-objective optimization algorithm using reference-point based nondominated sorting approach, Part II: handling constraints and extending to an adaptive approach, IEEE Transactions on Evolutionary Computation, 18 (2013) 602–622.
- [33] Y. Li, H. Liu, K. Xie, X. Yu, A method for distributing reference points uniformly along the Pareto front of DTLZ test functions in many-objective evolutionary optimization, 5th International Conference on Information Science and Technology (ICIST), (2015) 541–546.
- [34] A.B. Ruiz, R. Saborido, M. Luque, A preference-based evolutionary algorithm for multiobjective optimization: the weighting achievement scalarizing function genetic algorithm, Journal of Global Optimization, 62 (2015) 101– 129.
- [35] B. Srdjevic, Linking analytic hierarchy process and social choice methods to support group decision-making in water management, Decision Support Systems, 42 (2007) 2261– 2273.
- [36] U. Ebert, H. Welsch, Meaningful environmental indices: A social choice approach, Journal of Environmental Economics and Management, 47(2004) 270–283.
- [37]S.M. Ashrafi, A.B. Dariane, Performance evaluation of an improved harmony search algorithm for numerical optimization: Melody Search (MS), Engineering applications of artificial intelligence, 26 (2013) 1301–1321.
- [38]K.S. Lee, Z.W. Geem, A new meta-heuristic algorithm for continuous engineering optimization: Harmony search theory and practice, Computer Methods in Applied Mechanics and Engineering, 194 (2005) 3902–3933.

- [39] M. Mahdavi, M. Fesanghary, E. Damangir, An improved harmony search algorithm for solving optimization problems, Applied Mathematics and Computing, 188 (2007) 1567–1579.
- [40] M. Fesanghary, M. Mahdavi, M. Minary-Jolandan, Y. Alizadeh, Hybridizing harmony search algorithm with sequential quadratic programming for engineering optimization problems, Computer Methods in Applied Mechanics and Engineering, 197 (2008) 3080–3091.
- [41] J.C. de Borda, Mathematical derivation of an election system, Isis, 44(1/2) (1781) 42–51.
- [42] K.J. Arrow, Social choice and individual values, John Wiley & Sons, Inc. (1951).
- [43] A. D'Angelo, A. Eskandari, F. Szidarovszky, Social choice procedures in water resource management, Journal of Environmental Management, 52 (1998) 203-210.
- [44] J.N. Mordeson, D.S. Malik, T.D. Clark, Application of Fuzzy Logic to Social Choice Theory, Chapman and Hall/ CRC, (2015).
- [45] K.J. Arrow, A.K. Sen, K. Suzumura, Handbook of Social Choice and Welfare, Elsevier (2010).
- [46] D. Black, On the rationale of group decision making, Journal of Political Economy, 56 (1948).
- [47]K.I. Inada, Majority rule and rationality, Economic Theory, 2 (1970) 27-40.
- [48]S. Kant, S. Lee, A social choice approach to sustainable forest management: An analysis of multiple forest values in Northwestern Ontario, For. Policy Econ., 6 (2004) 215–227.
- [49] R.U. Goetz, Y. Martinez, J. Rodrigo, Water allocation by social choice rules: The case of sequential rules, Ecological Economics, 65 (2008) 304–314.
- [50]J. Malkevitch, Voting and Elections: Introduction, http:// www.ams.org/samplings/feature-column/fcarc-votingintroduction.
- [51]A.D. Taylor, Mathematics and politics, Springer-Verlag, New York (1995).
- [52] N. Srinivas, K. Deb, Muiltiobjective optimization using nondominated sorting in genetic algorithms, Evolutionary Computation, 2 (1995) 221-248.
- [53]K. Deb, S. Pratab, S. Agarwal, T. Meyarivan, A fast and elitist multiobjective genetic algorithm: NGSA-II, IEEE Transactions on Evolutionary Computation, 6 (2002) 182– 197.
- [54]J.L. Cohon, D.H. Marks, A review and evaluation of multiobjective programming techniques, Water Resources Research, 11(1975) 208–220.



