Improvement of Dynamic Behavior of Suspension Footbridges by Modification on Hangers' System Arrangement

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Abstract:
The hanger systems of the footbridges are used in two vertical and inclined forms. Both systems have their own advantages and disadvantages. The inclined hangers are more prone to slackness and fatigue phenomenon and are stressed too much. There is no much slackness, fatigue phenomenon and over-stress in the vertical system, but this system is more prone to vertical vibration at low frequencies than inclined ones. In recent years, a new modification has been made to eliminate deficiencies in the inclined hanger system. In the modified system, the slackness phenomenon has been removed completely and the force variations of two adjacent hangers have been reduced significantly. In this study, modeling and analysis of the footbridge was performed with CSI Bridge software and the disadvantages of the old modified hanger system are eliminated by proposing a new modified hanger system. A modal analysis was also carried out to compare the dynamic characteristic such as natural modes and frequencies on a footbridge with the vertical, inclined, old modified and new modified hanger systems. Results showed that the new modified hanger system was improved compared with the old one in the terms of vertical vibration mode so that the new system had no vertical frequency in the pedestrian vertical frequency range.

Keywords:
Vertical hanger, inclined hanger, modified hanger, lateral mode, vertical mode

1. Introduction
For the structural safety verification of footbridges, and for the comfort guarantee of its users, it is fundamental to consider the effect of human induced vibrations, particularly vibrations due to pedestrian traffic should be within acceptable limits for users.

Footbridges (such as Millennium Bridge, London; and Solférino Bridge, Paris) have proven sensitive to vibration caused by humans. In recent years, increasing vibration problems have shown that footbridges should no longer be designed exclusively for static loads. An important source of dynamic excitement on footbridges is pedestrian excitation. Footbridge vibration can lead to problems with serviceability, as it can affect pedestrians’ comfort and emotional reactions. Breakdown or even harm due to human actuated dynamic powers has happened very rarely [1].

Actually, footbridges intended for human occupants are susceptible to vibrations because of one or more natural frequencies inside the scope of typical human activities such as walking, running, bouncing or jumping and may suffer from severe serviceability problems with vibration, particularly in the lateral direction. The excessive lateral vibration of many footbridges around the world, such as the Millennium Bridge in London and the M - Bridge in Tokyo, has demonstrated this phenomenon.

Slim suspension footbridges always have four primary types of vibration modes: lateral, torsional, vertical and longitudinal modes. The lateral and torsional modes are frequently combined together and gotten to be two sorts of coupled modes: lateral-torsional modes or torsional-lateral modes. Such slim footbridges also have different lateral and vertical dynamic performances. Damping has noteworthy effect on the vertical vibration but only a small impact on the lateral one. Huang [2] investigated dynamic characteristics of slender suspension footbridges. Huang proposed a suspension footbridge model with pre-tensioned reverse profiled cables. Ivana Štimac Grandić [3] in a paper presented an extensive state of art in the field of pedestrian load models and vibration comfort criteria for pedestrian bridges. Samadi and Zamani Ahari [4] conducted a series of analysis on a suspension footbridge as a case study under both actual human loads and the simplified loads suggested by the code and the results were compared. They found out that in the same crowd loading, the actual human loading created greater vertical accelerations compare to EUR 23984 EN method results. Kratochvíl and Križan [5] studied the dependence of the increasing dynamic response on the number of synchronized pedestrians crossing the footbridge. They concluded that Synchronized group of people
going in the frequency identical to the natural frequency of structure could excite vibration of the structure in the corresponding mode shape.

Based on harmonic load models, Pedestrian effects are generally characterized. The first harmonic’s dominant contribution leads to the following critical range for natural frequencies $f_i$:

- $1.25$ Hz $\leq f_i \leq 2.3$ Hz (for vertical and longitudinal vibrations)
- $0.5$ Hz $\leq f_i \leq 1.2$ Hz (for lateral vibrations)

In some cases, natural frequencies lie in an interval susceptible of excitation by the second harmonic of pedestrian excitement. In these situations, if the effects of the second harmonic of pedestrian loads are considered relevant, the critical range expands to:

$1.25$ Hz $\leq f_i \leq 4.6$ Hz (for vertical and longitudinal vibrations)

Pedestrian bridges - with natural frequencies $f_i$ in the critical range – need to be the object of a dynamic evaluation to pedestrian excitation. The second harmonic of pedestrian loads does not affect lateral vibrations. (Table 1 shows the first and second harmonics of pedestrian loads)

The critical range of natural frequencies is primarily based on the empirical study of the step frequencies $f_s$ of pedestrians. In order to be consistent with the Eurocodes standards, the characteristic values $f_{s, 5\%}$, slow and $f_{s, 95\%}$, fast used are primarily based on the 5th and 95th percentile values [6].

Pedestrian suspension bridges may have inclined or vertical hanger systems that transfer forces from the deck to the main cables. Because of the damping role, inclined hangers act better than vertical ones against dynamic and lateral loads. However, inclined hangers require changes in their systems to achieve an optimum system due to the slacking under the excessive tension force and also due to early fatigue compared to vertical hangers [7].

For this reason, modification on the inclined hangers’ system was recommended by Barghian and Moghadasi for the first time to achieve an optimal system [8]. In the proposed modified hanger system, the slackness phenomenon was completely removed and force variations of two adjacent hangers were significantly reduced compared to the inclined ones. Also, Moghadasi et al. investigated a footbridge with the modified hanger system under human harmonic loads [9, 10]. Moghadasi and Moghadasi [11] in another paper analyzed a suspension footbridge with inclined hangers with two boundary conditions, once with a fixed support and another with a support relying on a soil material
in order to investigate soil effects on their structural responses because they consist of considerable flexibility and also geometrically nonlinear members such as main cables and hangers.

In order to modify the only inclined hangers, Moghadasi and Barghian added a link (member) between two adjacent inclined hangers in a case study footbridge (Soti Ghat footbridge in Nepal).

The new modified hanger system was studied based on removing slackness and overstress phenomenon by the authors. They formulated the length and the height of the added member between two adjacent inclined hangers in different footbridges [12]. In the following parts, the term of “old modification” refers to the Moghadasi’s and Barghian’s hangers’ modification; while the term of “new modification” refers to the Mehrgan’s and Barghian’s present hangers’ modification.

In this study, the advantages of the old modified hanger system were maintained by using a new modification on the hanger system, while its disadvantages were removed. The disadvantage of old modification was eliminated by considering modal analysis. In addition, the dynamic features on a footbridge with vertical, inclined, old modified and new modified hangers’ systems were compared. The results showed that in comparison with vertical, inclined and old modified hanger systems, the new modified hanger system was significantly improved.

2. Analytical model

In this paper, four suspension footbridges with vertical, inclined, old and new modified hanger systems were analyzed. As a case study, the data of the Soti Ghat pedestrian bridge in Nepal were used. Similar properties were used in four bridges. The spans were stiffened by two longitudinal pipe shaped beams. The diameter of the main cables was set to 120 mm and hangers to 26 mm. At every specified distance of the deck there was also transverse beam forming pinned connections between the longitudinal beams. The deck of each bridge was stiffened by two horizontal pipe shaped braces laterally. The towers comprised steel pipes, braced laterally by diagonal braces. In the bridge models, steel (with the Young modulus of $2\times10^{11}$ N/m² and the density of 7850 kg/m³) was chosen for all members. For main cables and hangers, the following values were used: $f_y = 1.18\times10^9$ N/m², $f_u = 1.57\times10^9$ N/m² and the density of 7850 kg/m³ where $f_y$ and $f_u$ are yield stress and tensile strength, respectively. The amount of pre-stressed load of cables was considered based on the weight of cables,
sag and axial stiffness in cables. The views of vertical and inclined hanger systems of the Soti Ghat Bridge are shown in Figures 1 and 2.

![Fig. 1: Soti Ghat pedestrian suspension bridge model with vertical hangers’ system [12]](image1)

![Fig. 2: Soti Ghat pedestrian suspension bridge with inclined hangers’ system [12]](image2)

2-1- Verification of the footbridge model

The Soti Ghat bridge was modeled by Barghian and Moghadasi [8] using SAP2000 software. They considered different load patterns to analyze the bridge statically. In order to verify the present model, the same load patterns were applied to the present model using CSI Bridge software and identical or very close results were obtained. Here, only two of the graphs are shown (Figures 3 ad 4).

![Fig. 3: The vertical displacements of the deck in footbridges with different hanger systems subjected to load pattern D](image3)
Fig. 4: The Shear force in stiffening beams in three hanger systems subjected to load pattern D

The error between the amount of curves in reference [8] and the present study was less than 5%.

2-2- Modifying the inclined hanger system arrangement to improve its efficiency

The vertical hangers have usually been used in most pedestrian bridges and a few of them have been built with inclined hangers. The new model of hanger systems has been presented to remove the defects of both vertical and inclined hangers. In this model a horizontal member is added between two adjacent inclined hangers as shown in Figure 5, so that the distribution of load between two adjacent hangers is done by the added member. The cross section and material used in the added member is the same as that used in hangers.

Fig. 5: The specification of modified hangers’ system [12]

The old modified hanger system of the Soti Ghat footbridge is shown in Figure 6.
In the old proposed modified system, the constant length and height of 1m and 2m (L=1m, H=2m), were respectively, employed for the added member. In this study the old modified system was improved with the changes in arrangement of the hangers and position of the added member, the length of the added member was set to 1m in accordance with the following relations:

According to Figure 7(a) for a pair of arbitrary adjacent inclined hangers, the following linear equation can be written for AC line.

\[ L = \frac{L_b \left( h - H \right)}{2h} \]

Eq. (1)

Where, H, L and L_b are the height of the added member, the length of the added member and the length of the beam between two adjacent hangers, respectively.

Substituting the value of the added member height that is half the corresponding adjacent height of the hangers, gives the equation:

\[ L = \frac{L_b}{2} \]

Eq. (2)

In order to apply the initial effect to the added member and to transfer the force between the hangers, the length obtained from Eq. (2) must be corrected. The minimum value for this correction is 20 cm.
for the Soti Ghat footbridge [12], in this study for a more accurate comparison, the value of 25 cm is considered.

\[
L = \frac{L_h}{2} - 0.25
\]

\[L = 1.0 \text{ m}\]

And height of the added member was set variable. So that, the center height of two adjacent hangers was employed as a unique height for each added member (\(L=1\text{m}, H=\text{variable}\)). Figure 7(b) shows the view of a pair of new modified hanger with the added member. The new modified hanger system is shown in Figure 8.

![Figure 7(b) shows the view of a pair of new modified hanger with the added member.](image)

Fig. 8: Soti Ghat pedestrian suspension bridge model with new modified hangers’ system

3. Harmonic load models

3-1- Equivalent number of pedestrians for streams

Synchronous excitation can be caused by the combination of high density of pedestrians and low natural frequencies within the frequency range of pacing rate. The crowded pedestrian loads were modeled as uniformly distributed loads on the entire bridge deck (simulating an equivalent number of pedestrians at fixed locations). For the modeling of a pedestrian stream consisting of \(n\) ‘random’ pedestrians, the idealized stream consisting of \(n^\prime\) perfectly synchronized pedestrians should be determined (see Figure 9). The two streams are supposed to cause the same effect on a structure, but the equivalent one can be modeled as a deterministic load.

In Figure 8, \(n^\prime\cdot Q\), and \(\Phi_i(x)\) are equivalent number of pedestrians on a loaded surface of the bridge deck, amplitudes of the loads, and vectors of modal displacements taken into consideration, respectively.
3.2- Application of load models

Harmonic load models were provided according to the certain traffic class. There are two different load models to calculate the response of the footbridge due to pedestrian streams depending on their density: the load model for density $d < 1 \, P / m^2$, and the load model for density $d \geq 1 \, P / m^2$, $d$, $P$, and $m^2$ are the density of pedestrians, number of pedestrians and unit of surface, respectively. Both load models share a uniformly distributed harmonic load of $P(t) [N/m^2]$ that represents the equivalent pedestrian stream. $P(t)$ can be calculated from:

$$P(t) = P \cdot \cos(2\pi f \cdot t) \cdot n' \cdot \psi$$

Eq. (4)

where $P(t) = P \cdot \cos(2\pi f \cdot t)$ is the harmonic load due to a single pedestrian [3], $P$ is the component of the force due to a single pedestrian with a walking step frequency $f_s$, which is assumed equal to the footbridge natural frequency under consideration, $n'$ is the equivalent number of pedestrians on the loaded surface $S$, where $S$ is the area of the loaded surface. Also, $\psi$ is the reduction coefficient taking into account the probability that the footfall frequency approaches the critical range of natural frequencies under consideration. The amplitude of the single pedestrian load $P$, equivalent number of pedestrians $n'$ (95th percentile) and reduction coefficient $\psi$'s are defined in Table 1 [6], considering the excitation in the first harmonic or second harmonic of the pedestrian load.

Table 1. Parameters for the load model of pedestrian traffic

<table>
<thead>
<tr>
<th>$P [N]$</th>
<th>Vertical</th>
<th>Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>280</td>
<td></td>
<td>35</td>
</tr>
<tr>
<td>Reduction coefficient $\psi$</td>
<td>Vertical</td>
<td>Lateral</td>
</tr>
</tbody>
</table>
Equivalent number $n'$ of pedestrians on the loaded surface for load model of:

If density $d < 1 \text{ P/m}^2: n' = \frac{10.8\sqrt{\xi n}}{S}$

If density $d \geq 1 \text{ P/m}^2: n' = \frac{1.85\sqrt{n}}{S}$

Where $\xi$ is the structural damping ratio and $n$ is the number of the pedestrians on the loaded surface $S$ ($n = S \cdot d$) where $d$ is the density of pedestrians on the deck.

In the numerical analysis, the Hilber–Hughes–Taylor method was used for the non-linear time history analysis under the walking dynamic loads. In Figure 10, the harmonic vertically loads of $P(t)$ (that may be vertically or laterally) was applied to the vertical hangers’ system for a particular mode shape.

In the case of torsional modes with several sags, the amplitude of the force must be of the shape shown in Figure 11. As mentioned earlier, it should be noted that suspension bridges always have four main types of vibration modes [13]: lateral, vertical, torsional, and longitudinal modes. However, numerical results show that the lateral modes and torsional modes do not always appear as pure lateral or torsional vibration modes. Most vertical vibration modes appear as pure vertical modes, without corresponding lateral or torsional ones. However it is possible to consider coupled lateral-torsional or torsional-lateral modes in order to considered harmonic loads, so to apply pedestrian loads in the case of coupled modes, the lateral and torsional loads have been placed simultaneously on the surface of bridges in order to the shape of lateral and torsional signs of coupled modes. Results show that the coupled lateral-torsional vibration modes are dominated by the lateral vibration modes in conjunction with the torsional vibration, while coupled torsional-lateral modes are dominated by torsional vibration modes. The loaded surface $S$ of the whole bridge deck should be considered with load acting up and down according to the investigated mode shape directions. The different load directions are simulating a phase shift of 180 (or $\pi$) for the pedestrians walking over the bridge. This can be
interpreted as full synchronization between every single pedestrian and the belly of the mode shape (direction), which he/she is reaching or just walking over. In numerical analysis, it was assumed that the load density was 1.5 person/m² (excessive live load on the deck) and the average weight of a person was 700 N [13].

![Fig. 10: Application of a vertically harmonic load according to mode shape](image)

![Fig.11: Sign of the amplitude of the load in the case of a torsion mode with several sags. Noted in red are the zones in which the amplitude of the load is positive, and in blue the zones in which the amplitude of the load is negative [13](image)

The following figures 12 and 13 illustrate the bending and torsion modes in vertical hangers’ system.
4. Results and Discussions

4.1- Lateral and vertical vibrations corresponding to the critical frequency range under the pedestrian load

Natural frequencies of the four bridges with different hanger systems were calculated. Results are shown in Tables 2 and 3. Dead load and pre-stressing load of cables were considered as an initial condition for the calculation of natural frequencies. The critical ranges for natural frequencies $f_i$ of footbridges with pedestrian excitation are as follows:

For vertical and longitudinal vibrations $1.25 \text{ Hz} \leq f_i \leq 4.6 \text{ Hz}$ and for lateral vibration $0.5 \text{ Hz} \leq f_i \leq 1.2 \text{ Hz}$.

According to the Tables 2 and 3, some modes of the footbridges with vertical, inclined, old and new modified hanger systems have coincided to the critical bandwidth of pedestrian frequencies for lateral and vertical vibrations, which means that they are prone for excitation by walking pedestrians.

The natural frequency variation diagram for four hanger systems is shown in Figure 14. It was realized that, up to the ninth mode, the dynamic behavior of the new modified system is somewhat identical to the vertical and old modified systems. After ninth mode, the frequencies of the new system are very close together.

The critical pedestrian frequencies of Tables 2 and 3 are summarized in Table 4 (only the values inside the bridge excitation frequency range are shown in Table 4). As it is seen from Table 4, the lateral vibration mode has been transferred from the third mode in the inclined system to the sixth and
forth modes in the old and new modified hanger systems, respectively. The new modified system was improved in comparison with old one in the terms of vertical vibration mode so that the new system has no vertical frequency in the pedestrian vertical frequency range.

Table 2. Natural modes and frequencies of the suspension footbridge with vertical and inclined hangers

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Natural Frequency (Hz)</th>
<th>Description of Mode Shape</th>
<th>Mode No.</th>
<th>Natural Frequency (Hz)</th>
<th>Description of Mode Shape</th>
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<td>Lateral</td>
</tr>
<tr>
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Table 3. Natural modes and frequencies of the suspension footbridge with old and new modified hanger systems

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Natural Frequency (Hz)</th>
<th>Description of Mode Shape</th>
<th>Mode No.</th>
<th>Natural Frequency (Hz)</th>
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Table 4. Natural frequencies and mode shapes of the different hanger systems

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<th>Mode Shape</th>
<th>Natural Frequency (Hz)</th>
<th>Mode Shape</th>
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<td>Vertical</td>
<td>-</td>
<td>-</td>
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</tbody>
</table>

4-2- Comparison of the hangers’ force oscillations in different hanger systems

The force oscillation of the hangers in the new modified system has been significantly reduced. As mentioned earlier, the positioning and connection of the added member between the two inclined adjacent hangers distributes the hanger force almost uniformly across the inclined hangers’ system and prevents excessive oscillations in this system.

Figures 15 and 16 show hangers’ force under self-weight of the bridge in vertical and new modified hanger systems in comparison with the inclined one.
As it is seen from Figure 16, the vertical hanger force is in the range of the upper and lower hangers' force fluctuations.

4-3- Comparison of lateral displacements of the bridges with different hanger systems

According to the Table 4, some modes of bridges with four hanger systems have coincided to the critical bandwidth of frequencies, which means that they are prone to excitation by walking pedestrians. The lateral modes with equal frequencies from four different bridges were chosen and compared. The applied harmonic loads on four systems of bridge hangers are given in Table 5. The indices of loads in Table 5 represent the modes which have lateral vibration shape with identical frequencies. Then the lateral displacements of the systems were compared.

Table 5. Pedestrian dynamic loads according to the lateral natural frequency for the footbridges

<table>
<thead>
<tr>
<th>Hanger Systems</th>
<th>$P_t(t)$: Dynamic Load (N/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>$P_v(t) = 5.6\cos(2\pi \times 0.784t)$</td>
</tr>
<tr>
<td>Inclined</td>
<td>$P_i(t) = 5.6\cos(2\pi \times 0.788t)$</td>
</tr>
<tr>
<td>Old modified</td>
<td>$P_o(t) = 5.6\cos(2\pi \times 0.787t)$</td>
</tr>
<tr>
<td>New modified</td>
<td>$P_n(t) = 5.6\cos(2\pi \times 0.784t)$</td>
</tr>
</tbody>
</table>
Fig. 17: Lateral displacement of the footbridge with different hanger systems

As is shown from Figure 17, the lateral displacements of the vertical, old and new modified hanger systems are identical and they are lower than the inclined one. It was realized that the inclined hanger system has less stiffness against lateral vibration comparing with the other three systems. (i.e. vertical, old and new modified systems)

4-4- Comparison of vertical displacements of the bridges with different hanger’ systems

To compare bridges vertical displacements, similar to lateral displacements, the modes were chosen that firstly, they had vertical mode vibration shapes and secondly, the frequencies of those modes were equal. According to Tables 2 and 3, it was observed that the footbridges had no identical frequency related to vertical mode shapes to compare with each other in terms of vertical displacement. For this reason, equal frequencies of four hanger systems were compared two by two.

In order to compare vertical displacement in bridges with different hanger systems, the constant value of 44.86 N was employed as the amplitude of the harmonic load (p, n', ψ). The mentioned value was calculated by assuming \( \psi = 1 \) for all bridges. i.e.:

\[
P(t) = p \cdot \cos(2\pi f, t) \cdot n' \psi
\]

\[
d \geq 1 \quad \frac{P}{m^2} : \quad n' = \frac{1.85\sqrt{n}}{S} = \frac{1.85\sqrt{(200 \times 1.5)}}{100 \times 2} = 0.16
\]

\[
p \cdot n' \psi = 280 \times 0.16 \times 1 = 44.86
\]
Fig. 18: Maximum vertical displacements due to P1 for the bridges with vertical and old modified hanger systems

Figure 18 shows that the old modified hangers’ system has fewer vertical displacements in comparison with the vertical hangers’ system.

Fig. 19: Maximum vertical displacements due to P10 and P16 for the bridges with inclined and old modified hangers’ systems, respectively

Fig. 20: Maximum vertical displacements due to P5 and P3 for the bridges with old and new modified hanger systems, respectively
Figure 20 shows that the new modified hangers’ system has fewer vertical displacements in comparison with the old modified hangers’ system. By comparing the Figures 18 to 20, it is realized that the new modified system has fewer vertical displacements than conventional hanger systems and acts better than the others.

5. Conclusion

- The number of critical pedestrian frequency in the vertical, inclined, old and new modified hanger systems are 3, 4, 2 and 1, respectively. The results showed that in terms of the number of excitation frequency within the critical frequency range for pedestrian induced dynamic loads, both the old and new modified systems have less excitation frequency than the vertical and inclined systems and finally the new modified system has the least excitation frequency.

- In the new modified hanger system in addition to the complete elimination of slackness phenomenon, unlike the old modified hanger system, the upper and lower sections forces of hangers were almost equal.

- Inclined hanger system has less stiffness against lateral vibration comparing with the other three vertical, old and new modified systems.

- The maximum amount of lateral displacement of the footbridge is related to the inclined system, which is 102.4%, 99.7% and 99.9% more than the vertical, old and new modified model, respectively. The lateral displacements in the three vertical, old and new modified systems are very close together and their variations are very small and less than 1%.

- The new modified hanger system has less vertical displacement in comparison with three other vertical, inclined and old modified hanger systems. The maximum displacement in the bridge with the old modified hanger system is 260.3% higher than the new modified one.

- The new modified system was improved in comparison with old one in terms of the vertical vibration mode so that the new system has no vertical frequency in the pedestrian vertical frequency range.
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References