Hydraulic jump in a rough sudden symmetric expansion channel

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Abstract:

Control of the high velocity and kinetic energy of the flow at downstream of the hydraulic structures in order to prevent the erosion of the channel bed is one of the most important concerns of hydraulic engineers. For this reason, energy dissipation structures are used. The sudden symmetrical expanding channel is an energy-dissipation structure that requires minimal tailwater depth for the formation of hydraulic jumps. Rough beds are important for reducing stilling basin dimensions and the effect of the roughness elements impact the hydraulic jump. The effect of bat-shaped elements in a sudden expansion channel have been investigated here. The experiments were performed in a rectangular channel with symmetric expansion ratios of 0.67, 0.5, and 0.33, and a Froude number that ranged from 4.6 to 11.3. The results show that the shear stress in a rough bed of a sudden expansion channel was more than 12 times greater than the shear stress in a smooth prismatic channel. Also, the secondary depth and S-jump length in a rough bed compared to the smooth bed decreased by 22% and 9-13%, respectively. Finally, several equations were developed to predict the hydraulic jump on the rough bed. The correlations have R^2 values of more than 0.988 and NRMSE values of less than 2.5%. These highly accurate equations are easy and simple to apply for the design of enlarged stilling basins.

Keywords:

Depth ratio, Energy dissipation, Hydraulic S-jump, Rough bed, sudden expansion
1. Introduction

Energy dissipation downstream of spillways, weirs, chutes, and gates, is necessary to prevent erosion and cavitation; it is an important task of hydraulic engineers [1, 2]. In fact, often, dissipation of the kinetic energy of the flow is the main purpose of a hydraulic structure. A stilling basin is one of the most common energy dissipation structures; it can form a hydraulic jump by transforming the flow regime from supercritical to subcritical [3, 4]. In recent years, many researchers have studied the classical hydraulic jump in a horizontal prismatic channel with a smooth bed [5-8]. Reference [5] compared a hydraulic jump with wall jets. Reference [6] examined the roller length of a classic hydraulic jump and provided estimating equations and [7] examined air entrainment in a hydraulic jump and reported the distribution of air concentration and the frequency of bubble formation.

The formation and stabilization of hydraulic jumps in stilling basins depend on the tailwater level. In cases where the tailwater level is low, an expansion section can be used to ensure jump formation [9-11]. Depending on the toe of the jump and the tailwater level, jumps in the sudden expansion channel are divided into three categories: a Repelled hydraulic jump (R-jump), a hydraulic jump (S-jump), and a Transitional hydraulic jump (T-jump) [9-12].

Extensive studies were carried out on the formation of a hydraulic jump in sudden expansion channels [9, 13-19]. Most of these studies investigated the depth ratio of the hydraulic jump in the sudden expansion and presented correlating relationships to predict it. Pagliara and Palermo (2009) investigated both the scour geometry and the hydraulic jump downstream of a block ramp in symmetrically expanding stilling basins. The results show that for the similar hydraulic and geometric conditions, the expansion causes an increase in the scour pit depth [20]. The effect of height and location of the sill on the characteristics of the S-jump in a sudden expansion channel with a sudden symmetric expansion was investigated by Zare and Doering (2011). Considering the dimensionless parameters of height and location of the sill, they showed that with an increase in these parameters, the secondary depth of the S-jump decreased [21]. Numerical study of symmetric submerged spatial hydraulic jumps (SSHJ) in a sudden expansion showed that the jump roller created near the lateral walls of the channel is stronger than in its center [22].

Stilling basins are effective for use as an energy dissipation structure, they are economical to implement. On the other hand, roughening the bed along with improving the jump characteristics is a means to reduce the dimensions of the stilling basin. Other measures may include the use of a chute block at the beginning, baffle blocks in middle and sill at the end of the jump. The main purpose of these measures is to reduce the jump length and dimensions of the basin [23, 24]. A hydraulic jump is used to prevent downstream channel erosion and excessive energy dissipation. Therefore, the jump location must be resistant to erosion and cavitation at high flow velocities. To prevent cavitation, the floor should either be smooth or the hydraulic jump controlling elements should be placed in a way so that their upper surface is equal to the upstream channel [25].

Several researchers have experimentally studied the characteristics of hydraulic jumps on a rough bed [25-33]. AboulAtta et al. (2011) examined hydraulic jumps on a rough bed with T-shaped roughness elements and different densities. They showed that T-shaped elements with a density of 8% have better results than those with rectangular elements with a density of 10% [26]. Parsamehr et al. [30] also examined the characteristics of hydraulic jumps on an adverse slope with a rough bed. They showed that the secondary depth and the length of the jump decreased with increased height of the roughness elements [30]. Norouzi
et al. [34] studied energy dissipation from an inclined drop with a screen. Results revealed that for a screen with a porosity of 50%, all jumps formed were type A and by increasing the drop height, the jumps formed behind the screen became a false jump type. For the higher porosity screen, the hydraulic jump decreased and the energy dissipation increased.

Neisi and Bajestan [35] were the first to examine the effect of a rough bed on the characteristics of an S-jump for Froude numbers ranging from 2-10 and expansion ratios of 1, 0.67, 0.5, and 0.33. Their results showed that the roughness of the bed decreased the secondary depth of the S-jump by ~16 to 20% [35]. Turkmenzad et al. [36] investigated spatial jumps in a sudden asymmetric expanding basin on a rough bed with two heights of roughness elements. They developed relationships to predict spatial jump characteristics [36].

An S-jump in a sudden expanding channel increases the jump length compared with the classical jump. On the other hand, the roughness of the bed improves the jump characteristics. Few studies were carried out on a rough bed of an S-jump. Hence, the purpose of this study is to investigate the characteristics of an S-jump using a new shape of roughness element and with different expansion ratios. Equations are developed for the shear stress coefficient, depth ratio, relative jump length, and jump efficiency as a function of Froude number and the ratio of upstream and downstream channel widths.

2. Materials and Methods

2.1. Experiments

The experiment was performed in a horizontal laboratory flume of 5 m length with a rectangular section of 30 cm width and 45 cm height in the Hydraulic Laboratory of Maragheh University, Maragheh, Iran. The wall and bottom of the flume were made of Plexiglas which allows for a detailed observation of flow phenomena. To create a Froude number range of 4 to 12, the height of the flume wall upstream of the gate was increased by 25 cm and a reservoir was created with an elevation of 70 cm at the beginning of the flume. A steel gate with a thickness of three millimeters was installed at the distance 50 cm from the upstream of the flume and the gate opening height was 1.3 cm for all models.

To create a symmetrical sudden expansion in the section, glass boxes of 5, 7.5, and 10 cm width (b1 = 10, 15, 20 cm), 20 cm height, and 50 cm length were used on both sides of the flume before and after the gate. Since the maximum length of the S-jump created on the smooth bed occurred with an expansion ratio of 0.33 and was equal to 1 m, the total length of rough bed was 1.2 m to ensure that it would accomodate the longest hydraulic jump. The roughness elements were made of black Plexiglas and had a bat shape. In a study by Parsamehr et al. (2017) [30], the roughness element density was 10% and roughness elements were distributed in a staggered arrangement as 7-6-7. A similar scenario was used in the present study. The longitudinal (T) and transverse (S) distances between the roughness elements within the basin were 2.87 and 2.2 cm, respectively. Figure 1 shows a schematic of the experiment model. The roughness elements are mounted on a glass pane and placed in such a way so that the crest of the elements were at the same level as the upstream and downstream beds of the basin. Downstream of the flume, a tailgate was used to form the jump.
In each series of experiments, the pump was initially switched on and the flow rate was adjusted using a rotameter installed on the pump. The rotameter is capable of measuring flowrate to ± 2%. The jump formation and position of its toe (at the beginning of expansion section to create S-jump) were set using the downstream tailgate. The initial depth, secondary depth, and jump length were measured in each experiment after stable flow conditions were achieved. The secondary depth and jump length were measured where the asymmetric flow was nearly horizontal and air bubbles were not present. The initial depth of the jump in the expansion section was measured using a point gauge with an accuracy of ±1 mm. The measurements were made at five location on a transverse section and the mean values were taken as the initial depth. For measurement of the secondary depth, the average depth of the five points of the transverse section was obtained. Then, using digital imaging of the jump length, the depth at the same section was estimated by the Get Data Graph Digitizer software. The mean depth values obtained from the Get Data Graph Digitizer and the mean value of depth measured by points gauge at five point of flume width were considered as the secondary depth of the jump. The jump length was also measured to an accuracy of ±1 mm. Figure 2 shows the top view of the S-jump occurring asymmetrically on the rough bed. The range of measured variables is also presented in Table 1.
Figure 2. Top and side view of an asymmetric S-jump on the rough bed

Table 1. Range of Measured Variables

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Bed type</th>
<th>Smooth bed</th>
<th>Rough bed</th>
</tr>
</thead>
<tbody>
<tr>
<td>b₁(cm)</td>
<td></td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>Q (L/min)</td>
<td></td>
<td>175-350</td>
<td>150-284</td>
</tr>
<tr>
<td>y₁(cm)</td>
<td></td>
<td>0.89-1</td>
<td>0.93-1.1</td>
</tr>
<tr>
<td>y₂(cm)</td>
<td></td>
<td>4.8-9.7</td>
<td>4.6-9</td>
</tr>
<tr>
<td>L₂(cm)</td>
<td></td>
<td>40-86</td>
<td>44-92</td>
</tr>
</tbody>
</table>

2.2. Dimensional Analysis

By considering Figure 1, the shear stress coefficient (ε) can be defined using the function of Eq. (1):

\[ \varepsilon = f_{1}(\rho, v_1, \mu, g, b_1, b_2, y_1, y_2, I, h) \]  

(1)

where \( \rho \) is the water density, \( v_1 \) represents the velocity at the toe of the jump, \( \mu \) denotes the dynamic viscosity, \( \varepsilon \) is the shear stress coefficient, \( g \) is the gravitational acceleration, and \( b_1 \) and \( b_2 \) represent the upstream and downstream widths of the channel, respectively. The term \( y_1 \) represents the initial depth of the jump, \( h \) is the height of the roughness element, and \( I \) is the density of roughness elements. By using the Pi-Buckingham theory and selecting \( \mu, v_1 \) and \( y_1 \) as the repeated parameters, the extracted dimensionless parameters are given in Eq. (2):
\( \varepsilon = f_2(Fr_1, Re_1) = \frac{v_1}{\sqrt{g y_1}}, Re_1 = \frac{\rho y_1 v_1}{\mu}, b_1 = B, \frac{h}{y_1}, I) \)  

(2)

Here, \( Fr_1 \) is the Froude number at the beginning of the S-jump, \( Re_1 \) is the inflow Reynolds number, \( h/y_1 \) is the dimensionless height of the roughness element, and \( B \) is the ratio of upstream and downstream channel widths (the expansion ratio). In the current study, the roughness density was a constant and the Reynolds number ranged from 13029 to 33875. Therefore, the flow was turbulent and this made it possible to ignore the effects of viscosity and roughness density. The range of initial depth variation was also insignificant because a constant gate opening height was used for the expansion ratio. On the other hand, given the fact that the height of the roughness element was constant, we were able to ignore the effect of the dimensionless height of the roughness element (2.8 \( \leq \frac{h}{y_1} \leq 3.14 \)). Hence,

\( \varepsilon = f_3(Fr_1, B) \)  

(3)

Similarly, the jump length \( (L_j) \), secondary depth \( (y_2) \) and energy loss \( (E_L) \) can also be expressed as follows:

\( y_2 = f_4(\rho, v_1, \mu, g, b_1, b_2, y_1, I, h) \)  

(4)

\( L_j = f_5(\rho, v_1, \mu, g, b_1, b_2, y_1, y_2, I, h) \)  

(5)

\( E_L \) or \( (E_1 - E_2) = f_6(\rho, v_1, \mu, g, h, b_1, y_1, E_1, I, h) \)  

(6)

where, \( E_1 \) and \( E_2 \) are the specific energy values upstream and downstream of the jump. Finally, the depth ratio \( (\frac{y_2}{y_1}) \), relative length \( (\frac{L_j}{y_1}) \) and efficiency of the jump \( (\frac{E_L}{E_1} = \eta) \) can be expressed by Eq. (7) as:

\( \frac{y_2}{y_1}, \frac{L_j}{y_1}, \eta = f_7(Fr_1, B) \)  

(7)

The ranges of initial Froude numbers and expansion ratios are presented in Table 2.

<table>
<thead>
<tr>
<th>Independent parameters</th>
<th>expansion ratio (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.67</td>
</tr>
<tr>
<td>initial Froude number ( (Fr_1) )</td>
<td>4.6-11.1</td>
</tr>
</tbody>
</table>

2.3. Theoretical Background

The shear stress at the bed of a hydraulic jump is determined using the momentum equation. By applying the momentum equation in a sudden expansion section with a rough bed and selecting \( F_\tau \) as the resultant shear force along with the jump yields (see Fig. 1).

\[ F_\tau = (M_1 - M_2) + (F_1 - F_2) + F_E \]  

(8)
where, \( F_1 \) and \( F_2 \) are the pressure forces before and after the jump, \( M_1 \) and \( M_2 \) are the momentum forces before and after the jump, \( F_E = 0.5 \gamma (b_2 - b_1)^2 y_1^2 \) is the pressure force on the expansion side wall, \( \gamma \) represents the density of the water, and \( b_1 \) and \( b_2 \) are the upstream and downstream widths of the channel. The shear stress coefficient (\( \varepsilon \)) in a smooth and rough bed can be calculated by Eq. (9) [25, 30, 35, 36].

\[
\varepsilon = \frac{F_1}{0.5 \gamma y_1^2} \tag{9}
\]

Equation (3) is proposed for calculating the shear stress coefficient in a smooth bed, as follows from [25]:

\[
\varepsilon = 0.16 \text{Fr}_1^2 - 0.8 \text{Fr}_1 + 1 \tag{10}
\]

Here, \( \text{Fr}_1 \) is the initial Froude number. In the current study, the shear stress coefficient in the sudden expansion channel was determined by Eq. (9) and the shear stress coefficient for a prismatic channel with a smooth bed was determined by Eq. (10).

### 2.4. Model Evaluation Criteria

An iterative method was used for calculating S-jump characteristics on a rough bed and statistical indices were used to quantify the ability of the correlation for performing calculations. For the indices, the coefficient of determination (\( R^2 \)) and the normalized root mean square error (NRMSE) were employed. The equations for calculating the statistical indicators are:

\[
R^2 = \left( \frac{n \sum (X_{\text{exp}} - X_{\text{cal}})^2 - \left( \sum X_{\text{exp}} \right)^2 \left( \sum X_{\text{cal}} \right)^2}{n(\sum X_{\text{exp}})^2 - (\sum X_{\text{exp}})^2} \right)^{1/2} \tag{11}
\]

\[
NRMSE = \left[ \frac{1}{n} \sum \left( X_{\text{exp}} - X_{\text{cal}} \right)^2 \right] \times 100 \times \frac{1}{M} \tag{12}
\]

Where \( X_{\text{exp}} \) is the experimental value, \( X_{\text{cal}} \) is the calculated value, \( n \) is the number of data sets, and \( M \) is the mean value. Values of \( R^2 \) near one and NRMSE less than 10% are considered acceptable [37].

### 3. Results and Discussion

A total of 40 different experiments were carried out on the physical model with smooth and rough beds and resulting values of shear stress coefficient, depth ratio, relative jump length, and jump efficiency were found. A discussion of these results will follow.

#### 3.1. Shear stress coefficient

One of the main reasons causing the reduction of the secondary depth and jump length on a rough bed compared to the smooth bed is due to the increase in the shear stress on the bed. Therefore, evaluating the shear stress and the shear stress coefficient is important. The variation of the shear stress coefficient in a sudden expansion channel with a rough bed for various Froude numbers and expansion ratios is shown in Fig. 3. As seen, the increase in the shear stress coefficient of the bed is proportional to the increase in the
initial Froude number. The shear stress coefficient increases with a decrease in the expansion ratio of the section. The shear stress coefficients are very close to each other and the expansion effect is small. The reason for this behavior is related to the passage of flow lines through roughness elements which cause these lines to collapse, resulting in a vortex between the elements. The shear stress at the bed increases and the flow becomes turbulent with the increase in the Froude number and vortices are generated behind the various elements (see Fig. 1).

Figure 3. Shear stress coefficient versus Froude number and expansion ratio

The shear stress coefficients for the sudden expansion channel with expansion ratios of 0.67, 0.5, and 0.33 were 12.86, 13.3, and 14, respectively. Considering the experimental data of the present study, Eq. (13), with $R^2$ of 0.997 and NRMSE of 2.5%, is proposed to predict the shear stress coefficient in the rough sudden expansion channel.

$$\varepsilon = 0.687 F_{r1}^{2.18} - 8.06B$$

(13)

3.2. Depth Ratio
Dimensions of the stilling basin depend on the secondary depth. Consequently, knowledge of the jump is of particular importance for the design of basins. Based the dimensional analysis, the depth ratio of the S-jump was calculated using the initial Froude number and the expansion ratio. Figure 4 shows the depth ratio of the S-jump on smooth and rough beds as well as the depth ratio computed by the Belanger equation for a prismatic channel and the depth ratio of the S-jump from Neisi and Shafai Bajestan [35]. As seen in Fig. 4a, for all divergent ratios and Froude numbers, the depth ratios of the S-jumps on rough and smooth beds are less than that for the prismatic channel. Also, the depth ratio of the S-jump depth is directly related to the Froude number and expansion ratio. Therefore, an increase in the Froude number causes an increase in the depth ratio while a decrease in the expansion ratio reduces the depth ratio. Taking Figs. 4b, 4c, and 4d into consideration, and by comparing the depth ratios on the rough bed of the present study to those of the smooth bed of Neisi and Shafai Bajestan [35], it can be seen that the depth ratios of this study are low.
main reason for this difference is the dimensionless height of the roughness element and the different element shapes. Compared to the smooth bed, the depth ratio of S-jump for all expansion ratios decreased by about 22%.

Figure 4. Depth ratios of a S-jump for different expansion ratios, (a) all expansion ratios, (b) $B = 0.67$, (c) $B = 0.5$, (d) $B = 0.33$

The relationship between the depth ratio as a function of the initial Froude number and the expansion ratio with $R^2$ of 0.988 and NRMSE of 2.4%, is obtained as follows:

$$\frac{y_2}{y_1} = 0.6165 Fr_1 + 2.65 LnB + 2.28$$

Next, the secondary depth of the jump in the sudden expansion channel of the present study is compared with results of Neisi and Shafai Bajestan [35] for the secondary depth of a classical jump in a prismatic channel. The dimensionless depth deficit parameter (D) was calculated according to Ead and Rajaratnam [25]:

$$D(\%) = \frac{y^* - y_2}{y^*} \times 100$$
Where $y^*$ represents the secondary depth of the jump derived from the Belanger equation (a classical jump) and $y_2$ is the secondary depth of the S-jump on the rough and smooth beds. The average values of the dimensionless depth deficit parameter are given in Fig. 5.

![Figure 5. Average values of depth deficit parameter](image)

As seen in Fig. 5, sudden expansion channels with a rough bed are compared with a classical jump. The secondary depth of the jump for expansion ratios of 0.67, 0.5 and 0.33, respectively, decreased by 43.5, 49 and 59.3%.

### 3.3. Relative Jump Length

The relative length of the S-jump versus Froude number for expansion ratios of 0.67, 0.5 and 0.33 are shown in Fig. 6. The relative lengths of the classical jump in Fig. 6 were determined using Eq. (16) given by the Brablies and Peterka [38] or USBR.

\[
\frac{L_j}{y_1} = 220 \tanh\left(\frac{Fr_1 - 1}{22}\right)
\]

(16)
As shown in Fig. 6, the relative S-jump length in a sudden expansion channel increases in comparison with a prismatic channel; this trend increases with increasing the Froude number and reducing the expansion ratio. The average increases in jump length for expansion ratios of 0.67, 0.5 and 0.33 are 5.3, 6.2 and 9%, respectively. The collision of the jet onto the channel sidewall in the S-jump is one reason for the increasing jump length. The effect of this collision decreases with decreasing expansion ratio of the section. According to Fig. 6, the roughness of the bed decreases the S-jump length compared to the smooth bed and also reduces the intensity of the jet collision onto the channel wall. By increasing the Froude number for all expansion ratios, the S-jump length values on the rough bed decrease compared to the prismatic and sudden expansion channel with a smooth bed. For Froude numbers greater than 7, the jump length is less than the corresponding value in a classic jump. For example; with expansion ratios of 0.67, 0.5, and 0.33, the jump length decreases by 5, 6.4, and 10.5%, respectively. The average reduction in the length of the S-jump on the rough bed at all expansion ratios ranges from 9-13.2% relative to a smooth bed.

Equation (17), with $R^2$ of 0.991 and NRMSE of 2.2%, can be used to predict the relative length of the S-jump on the rough bed:

$$\frac{L_j}{y_1} = 7.3F_{r1} - 5.74LnB + 1.72$$

(17)

3.4. Efficiency of Jump

The difference between upstream and downstream specific flow energy, divided by the upstream value, is called the jump efficiency or relative energy loss. Figure 7 shows values of the jump efficiency from the present study with smooth and rough beds compared with those obtained by Neisi and Bajestan [35]. The values for the classical jump efficiency in a prismatic channel with a smooth bed were determined according to the relation $\eta = (1 - \sqrt{2} / F_{r1})^2$ [39].
Figure 7. S-jump efficiency versus Froude number for sudden expansion channels. (a) $B = 0.67$, (b) $B = 0.5$, (c) $B = 0.33$

In Fig. 7, the S-jump efficiency on smooth and rough beds is seen to increase with increasing the Froude number and decreasing expansion ratio. The expansion ratio of 0.33 is the highest value of jump efficiency. It can be also seen from Fig. 7 that the bed roughness increases the S-jump efficiency for all expansion ratios. According to Fig. 7, According to Fig. 7, the S-jump efficiency on the rough bed of the present study is higher than that of [35] and It seems that by decreasing of expansion ratio and increasing Froude number, the proximity of the jump efficiency of present study and [35] increase. According to values listed in Table 3, the sudden expansion channels with a smooth bed cause an increase of 12 to 24.8% in the jump efficiency compared to the prismatic channel situation. Also, by comparing rough and smooth beds, it is concluded that the effect of a rough bed in a sudden expansion channel is less than the effect of an expansion on jump efficiency. The rough bed increases S-jump efficiency by less than 7%. It can be seen that an S-jump on a rough bed with an expansion ratio of 0.33 leads to a 30% increase in jump efficiency compared to a classical jump in a prismatic channel with a smooth bed.
Table 3. Average values of efficiency increase in sudden expansion channel with smooth and rough bed compared to a prismatic channel

<table>
<thead>
<tr>
<th>Researcher</th>
<th>B</th>
<th>$G(%) = \frac{\eta - \eta^<em>}{\eta^</em>} \times 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This study (smooth bed)</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>This study (Rough bed)</td>
<td>0.67</td>
<td>19.2</td>
</tr>
<tr>
<td>Neisi and Shafai Bajestan (2013) [35]</td>
<td></td>
<td>17.57</td>
</tr>
<tr>
<td>This study (smooth bed)</td>
<td></td>
<td>16.3</td>
</tr>
<tr>
<td>This study (Rough bed)</td>
<td>0.50</td>
<td>23</td>
</tr>
<tr>
<td>This study (smooth bed)</td>
<td></td>
<td>24.8</td>
</tr>
<tr>
<td>This study (Rough bed)</td>
<td>0.33</td>
<td>30</td>
</tr>
<tr>
<td>Neisi and Shafai Bajestan (2013) [35]</td>
<td></td>
<td>26.64</td>
</tr>
</tbody>
</table>

In Table 3, $\eta$ represents the S-jump efficiency on a rough bed and $\eta^*$ is the classical jump efficiency in a prismatic channel. Equation (18) with $R^2$ of 0.974 and NRMSE of 1.4% was obtained through statistical analysis performed on the experimental data.

$$\eta = 0.23 \ln Fr - 0.067 \ln B + 0.29$$

4. Conclusions

In this study, the effect of bed roughness on a S-jump in sudden expansion channels was determined. A total of 40 different experiments were conducted on physical models with different expansion ratios. Parameters of shear stress coefficient, depth ratio, relative length, and S-jump efficiency were investigated and predictive equations were developed as a function of Froude number and the ratio of upstream and downstream channel widths.

The following conclusions were drawn:

1. The shear stress coefficient for the rough sudden expansion channel is 12 times greater than that for the smooth prismatic channel.

2. An increase in the shear stress reduces the conjugated depth of an S-jump by about 22%, compared with a smooth bed.

3. For an expansion ratio of 0.33, the sudden expansion channels with a rough bed reduce the conjugated depth by an average of 58% compared with a prismatic channel with a smooth bed.

4. The relative length of the S-jump on a rough bed for all expansion ratios was reduced by an average of 9 to 13%, compared with a smooth bed.
5. For Froude numbers greater than 7, the jump length is less than the corresponding value for a classic jump. The expansion ratio of 0.67, 0.5, and 0.33 decreased the length by 5, 6.4, and 10.5% compared to the classic jump, respectively.

6. Sudden expansion channels with a rough bed for all expansion ratios increased the efficiency of the S-jump by 13 to 23.5%, compared to classical jump in prismatic channels.

7. Considering that sudden expansion channels with rough beds have lower secondary depths and jump lengths than the corresponding values in a classic jump, they can be a suitable alternative for the standard USBR basins, as needed.

8. Bed roughness increases bed shear stress and turbulence in sudden expansion channels, reduces the intensity of the impact of the inlet jet with the sidewall of the channel in S-jump. Over time this can erode the channel wall.

Nomenclature

\[ B = \text{expansion ratios (\text{-})} \]
\[ b_1 = \text{upstream width of channel (m)} \]
\[ b_2 = \text{downstream width of channel (m)} \]
\[ D = \text{dimensionless depth deficit parameter (\text{-})} \]
\[ E_1 = \text{specific energy in upstream of jump (m)} \]
\[ E_2 = \text{specific energy in downstream of jump (m)} \]
\[ E_L = \text{energy loss (m)} \]
\[ F_{1I} = \text{inflow Froude number (\text{-})} \]
\[ F_1 = \text{pressure forces before jump (kN)} \]
\[ F_2 = \text{pressure forces after jump (kN)} \]
\[ F_{E} = \text{pressure force on expansion side wall (kN)} \]
\[ I = \text{density of roughness elements (\text{-})} \]
\[ L_j = \text{jump length (m)} \]
\[ M_1 = \text{momentum force before jump (kN)} \]
\[ M_2 = \text{momentum force after jump (kN)} \]
\[ v_1 = \text{velocity at toe of jump (m s}^{-1}\text{)} \]
\[ y_1 = \text{initial depths of jump (m)} \]
\[ y_2 = \text{secondary depths of jump (m)} \]
\[ y^* = \text{secondary depth of jump derived from Belanger equation (m)} \]
\[ E_F = \text{pressure force on expansion side wall (kN)} \]
\[ \gamma = \text{Specific weight of water (kN m}^{-3}\text{)} \]
\[ \varepsilon = \text{Shear stress coefficient (\text{-})} \]
\[ F_s = \text{shear forces along jump (kN)} \quad \eta = \text{jump efficiency (-)} \]
\[ g = \text{Gravitational acceleration (m s}^{-2} ) \quad \mu = \text{dynamic viscosity (Pa s)} \]
\[ h = \text{height of roughness element (m)} \quad \rho = \text{Mass density of water (kg m}^{-3} ) \]

References

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