A Multi-Period Multi-Objective Routing-Locating of Heterogeneous Vehicles (MPMORLHVP) planning Model for two echelon supply chain with Facility Breakdown

Danial Goran[1] , Amir Izadi[2*]

[1]Ph.d student, Department of Civil Engineering, Shomal University, Amol, Iran, (E-mail: Danial.goran1393@gmail.com)

[2]Assistant Professor, Department of Civil Engineering, Shomal University, Amol, Iran, (E-mail: Amirizadi@shomal.ac.ir)

[*]CORESPONDENCE amir izadi, Department of Civil Engineering, Shomal University, Amol, Iran

Abstract

Making decisions for allocating locations and determining the optimal route for vehicles will result in saving the number of transportation costs. In this paper, A Multi-Period Multi-Objective Routing-Locating of Heterogeneous Vehicles (MPMORLHVP) is proposed for determining the route and allocating the visit location for heterogeneous vehicles. The MPMORLHVP model can help optimize the routing and locating decisions for heterogeneous vehicles in a multi-period setting. It aims to find the most efficient and effective transportation plan, considering the specific context of a two-echelon supply chain and the possibility of facility breakdowns. Therefore main contribution of current study is provide a strategy for transporting heterogeneous vehicles over periods of time for handling and distribution in a sustainable supply chain. For this purpose, presented a multi objective mixed integer linear programming (MOMIP) that two objective functions are formulated to improve efficiency and effectiveness. The first objective is to minimize the total cost per path. The second goal is to minimize the total repair time of vehicles to visit all areas. The Epsilon Constraint (EC) method has been used to solve the proposed model. The applicability of the proposed model is shown via a numerical problem. The results obtained from solving the proposed model are compared with the routing plan. Based on the obtained results, the lowest allocation cost and duration of vehicle repairs have been calculated separately in each period. In the first period, the lowest and in the second period, the highest amount of cost has been calculated. In addition, in the second period, the lowest and in the third period, the maximum service time of vehicles has been determined. In addition, the results of this study can provide an advantage to decision-makers so that they consider appropriate strategies for disaster response.

Keywords: **Disaster, Allocation, Routing, Epsilon constraint**

nomenclature

Collections

K: The number of vehicles of type k

N: Number of vehicles of type n,

L: The number of service workers to carry out repairs on vehicles k and n

Parameters

c_{ijt}: Current expenditures on the route i to j in period t

t_{ijt}: The duration of movement in the route i to j in the period t

m_{it}: Demand at node i in period$t$

q_k: Vehicle capacity k

q_n: Vehicle capacity n

e_i: The earliest possible arrival time at node i

l_i: The latest time at node i

f_{it}: Vehicle repair time at node i in period t

r_{kt}: The maximum time of the route that vehicle k is allowed to move in period t

r_{nt}: The maximum time of the route that vehicle n is allowed to move in period t

cr_{it}: The number of vehicle breakdowns at node i in period t

c'_{ijt}: Service cost on the route i to j in period t

Variables

The variables in this research are classified into two categories: continuous variables and integer variables. Continuous variables are:

T_i: Time to enter node i

w_{it}: The waiting time for vehicle repair at node i in period$t$. The w_i variable is important for us since in this research, we want to take into account the risk of facility failure.

Also, binary variables are in this research follow as:

x_{ijkt}: It is a binary variable that can be assigned a value of zero or one. If x_{ijkt} is equal to one, the route from node i to node j is determined by vehicle k in period$t$. Otherwise, its value is equal to zero.

x_{ijnt}: It is a binary variable that can be assigned a value of zero or one. If $x_{ijnt}$is equal to one, the route from node i to node j is determined by vehicle n in period$t$. Otherwise, its value is equal to zero.

x_{ijklt}: It is a binary variable that can be assigned a value of zero or one. If $x_{ijklt}$is equal to one, on the route from node i to node$j$, vehicle k needs repair in period$t$. Otherwise, its value is equal to zero.

x_{ijnlt}: It is a binary variable that can be assigned a value of zero or one. If $x_{ijnlt}$is equal to one, on the route from node i to node j, vehicle n needs repair in period$t$. Otherwise, its value is equal to zero.

1. Introduction

Supply chain management (SCM) is traditionally defined as planning, executing, and controlling the process of delivering products from suppliers to customers to improve efficiency. In recent years, the transportation industry, with the rapid increase of the urban population, to move urban cargo is developing quickly, and the continued rise of the volume of road transportation has become inevitable [1]. According to the movement of transportation and road load which is responsible for traffic congestion and air pollution in cities, this phenomenon makes it vital to pay special attention to the environmental damage resulting from it in SCM[2]. Due to social, environmental, and economic constraints, modern SCM faces several challenges in designing an efficient distribution network with the lowest operational expense and impact on the environment[3]. Design distribution networks include two key components: determining the solution of distribution facilities and the way of communication between suppliers and customers. Hence, optimizing facility locations and vehicle routing decisions simultaneously is an important practical issue in SCM [4]. Based on the existing literature, the combining facility location problem and vehicle routing problem is defined as a location routing problem. In most studies, two level Locating Routing Problem (LRP) is usually verified through a single-period optimization model to find the optimal solution and achieve cost minimization or transportation efficiency maximization [5]. Furthermore, due to setup time and construction expense, facility location decisions are generally quite stable over time, whereas routing decisions change more than do facility location decisions[6]. However, in the long run, fixed location decisions can trigger additional operational costs, given that the spatial distribution of customer demands alters over time. In other words, the planning horizons of facility location and vehicle routing usually do not match in a single-period LRP [7]. Thus, considering numerous service periods in the whole planning horizon, a two-echelon multi-period location routing problem (MP-2E-LRP) is suggested to find optimal solutions, including periodic decision-making on facility location and vehicle routes[8]. MP-2E-LRP is a more realistic and practical location problem, compared to the single-period LRP framework, that can provide periodic location decisions in the long term, and routing plans in the short term[9]. Thereby, in addition to minimizing the total cost as the main objective of LRP, the lack of vital attention of LRPs to minimize the number of vehicles in a distribution network to reduce the impact on the environment and prevent traffic congestion is considered as well. From this perspective, paying attention to the sustainable logistics view provides an effective approach to improving the efficiency and sustainability of the distribution network. Therefore, an MP-2E-LRP considering the aspects of sustainability in the transportation system to design an efficient and sustainable distribution network, which seeks to simultaneously minimize the operational cost and the number of vehicles is proposed. Besides, multiple periods for allocate the transportation resources in the mathematical model for a two-level logistics network can be considered. For this purpose, in this research, 2-echelon multi period multi objective routing locating heterogeneous vehicle (2E-MPMORLHVP) are optimized, and a transportation strategy is considered in time horizon for the 2E-MPMORLHVP problem. Therefore, a multi-objective mathematical model that simultaneously minimizes the total cost and repair time of vehicles in case of failure is developed to model the proposed 2E-MPMORLHVP problem.

In summary, the current research is important as it offers a mathematical model that addresses the complexities of multi-period, multi objective routing and locating with facility breakdowns. It provides decision-makers with valuable tools to improve efficiency, make optimal decisions, manage breakdowns, and adapt to the challenges of modern transportation systems. Also, it can efficient routing and locating of vehicles can result in significant cost savings by minimizing fuel consumption, reducing vehicle idle time, and optimizing resource utilization. The research provides a mathematical framework to achieve this efficiency by considering multiple objectives simultaneously. Because, facility breakdowns are inevitable in real-world operations, and their impact on the routing and locating of vehicles can disrupt operations and increase costs. Considering incorporates facility breakdowns into the model, enabling managers to develop proactive strategies for minimizing the impact of breakdowns and effectively managing them when they occur. Therefore, proposed model helps decision-makers make more informed and optimized decisions by considering multiple objectives and constraints.

According to above mentioned, the most important objective of this research is:

- Provide a strategy for transporting heterogeneous vehicles over periods of time for handling and distribution in a supply chain.

Therefore, main research question is as follow:

- How to provide a strategy for transporting heterogeneous vehicles over periods of time for handling and distribution in a supply chain?

Also, the most important contributions of this research is follow as:

- Proposes a two-level multi period logistics network based on the periodic characteristics of logistics facilities and customers to obtain facility location decisions and vehicle routes instead of independent operations among several logistics facilities.
- Considering the heterogeneous transportation resource strategy in the proposed 2E-MPMORLHVP problem to optimize facility location decisions and vehicle schedules, thereby improving transportation efficiency and achieving heterogeneous resource configuration among multiple logistics facilities in the period Different times are provided.
- Present a bi-objective mathematical planning model considering different time periods, heterogeneous transportation resources, and transportation requirements between locations to minimize the total logistics operation cost and the number of vehicles in a considered two-level multi period distribution network.

In Figure 1, conceptual modelling of proposed model framework is depicted.

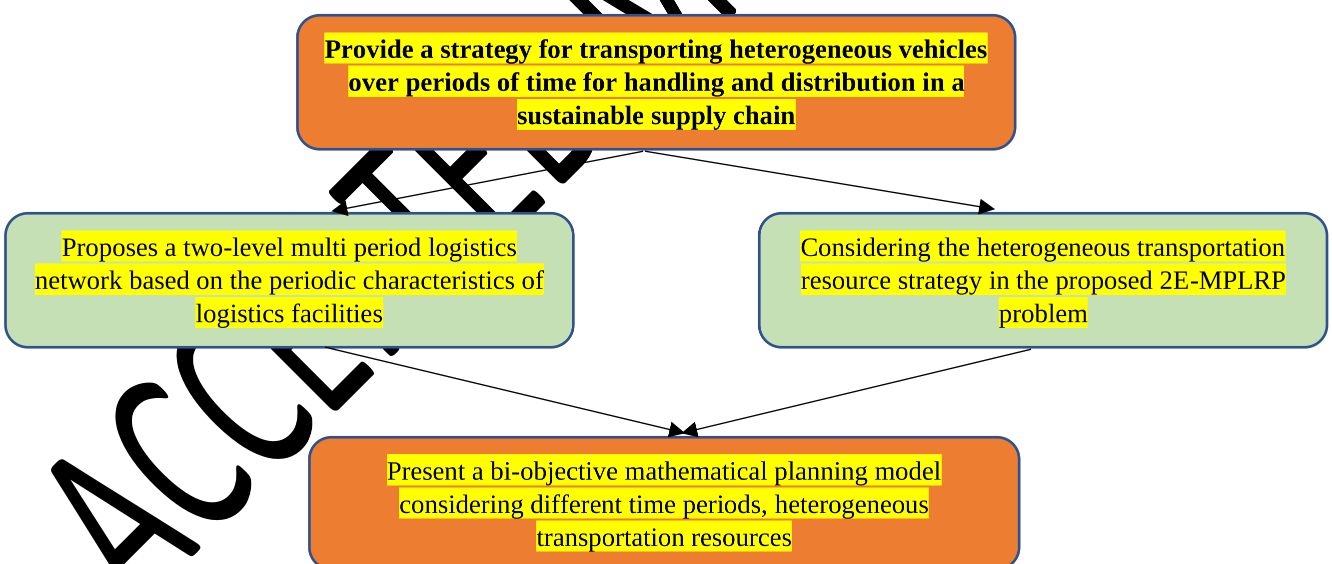

Figure1. Contribution

The rest of the article has been organized as it will be determined. In the second part, a literature review of past studies on the research topic has been presented. In the third part, the method of solving the research, including modeling and related details, has been presented. In the fourth part, the research findings from solving the proposed model have been presented. Finally, in the fifth section, a general conclusion has been presented.

2. Literature Review

Recently, two-level distribution systems are considered one of the most significant logistics issues in the location routing problem, which has been utilized in several scientific studies. The problem of two-echelon location routing problem in cargo distribution occurs once the goods of different origins are delivered to their respective destinations through intermediate facilities. For an instance, Yu et al [10] have developed a model for a multi-objective two-echelon location routing problem (2E-LRP) for scheduling garbage collection. In addition, to solve the model, an NSGA-II algorithm with directed local search has been introduced. Cheng et al[11] have presented a model in their study to minimize the cost and duration of cleaning up debris from natural disasters by considering the use of temporary sites for natural disaster debris management. The problem developed in this study is a two-stage multi-period location routing problem (MP-LRP) in which the main decisions of the location of temporary waste management sites and vehicle routing are made at both levels. For this purpose, a mixed integer programming is presented and a genetic algorithm is proposed to solve it. Wang et al[12] have presented a two-echelon multi-period location routing problem (MP-2ELRP), including facility location selection and vehicle routing optimization in two levels. To solve the problem, a two-stage hybrid algorithm, including k-means clustering and an improved multi-objective particle swarm optimization algorithm has been proposed. The k-means clustering algorithm has been used to allocate customers to distribution centers to receive services in multiple periods, and the particle swarm optimization algorithm has been used to determine vehicle routes and find Pareto optimal solutions. Santos-Gallara et al[13] introduce a generalized 2E-LRP model with two-dimensional loading constraints for the two-level location problem in their study. To solve the proposed model, they introduce an innovative scenario-based optimization method for different loading and evaluate its performance on real examples. Fallah-Tafti et al[14] introduce a multi-objective two-echelon location routing model (MO-2ELRP) in their study for moving cash to reduce the risk of theft in transportation. For this purpose, the amount of money carried by the vehicle has been used as a function of risk as the first objective and also the duration of money transfers as the second objective. To solve the problem, several precise and meta-heuristic methods in small to medium dimensions have been used. Cao et al[15] have introduced a two-level biomass resources routing location problem (2E-BLRP). Considering the predetermined supply of biomass resources, a mixed integer programming model is presented for the proposed model. The proposed model can determine the best locations for biomass collection facilities and the corresponding vehicle routes. To solve the problem, a hybrid heuristic algorithm based on neighborhood search and Tabu search has been introduced. Mohammad et al[16] investigated a distribution network design problem under uncertainty at strategic levels. In this study, a problem is defined as two-echelon stochastic multi-period capacity location routing problem (2E-SMCLRP). For this purpose, the network is divided into two capacitated distribution levels, each of which contains a specific location-allocation-transport scheme that must deal with future demand. The proposed model is modeled using a two-stage stochastic integer programming, where the first stage includes location and capacity decisions in each period in the planning horizon while routing decisions are determined in the second stage. An approach based on Benders analysis is proposed to solve the proposed problem. Xiao et al[17] introduce a two-echelon dynamic vehicle routing problem with active satellite stations (2E-DVRP-PSSs) that can optimize operational cost and construction costs. Do et al[18] developed a new joint delivery model to decrease operational costs and carbon emissions by enhancing collaboration and resource sharing. For this purpose, a mathematical model of multi-warehouse two-stage joint delivery (JD) location routing problem (MW-2E-JDLRP) is proposed for JD considering multiple objectives. Also, a hybrid heuristic algorithm is proposed to solve the proposed model. Heidari et al[19] propose a bi-objective mathematical programming model for a green two-echelon closed and open location routing problem (G-2ECOLRP), which includes two levels including factories, warehouses, and customers, to minimize costs and CO_2 emissions. The proposed model can determine the optimal routes, the number of optimal vehicles, and the location of facilities. To solve the proposed model, the modified epsilon method has been utilized to accurately solve the problem in small dimensions. In addition, NSGA-II meta-heuristic algorithm has been used. Another type of LRP problem that has attracted the researchers' attention is the vehicle routing problem (VRP). The VRP problem is one of the important problems in supply chain management, in which several vehicles concentrated in one or more locations (warehouses or nodes) must visit several customers each of whom has a specific request, and provide them with a service. This problem attempts to use mathematical models and route optimization in such a way that the distance traveled, the total travel time, the number of means of transportation, overdue fines, and finally the function of transportation cost are minimized and as a result Customer satisfaction reaches its maximum value. To exemplify, Nira et al[20] presented an integer programming model for a multi-trip vehicle routing problem with a time window, and loading time that depends on service type, and the travel duration (MTVRPTW-SDLT). The first model presented in this paper models the return of the vehicle to the warehouse. A deterministic method is used to solve the problem. Huang et al[21], have introduced a model for multi-trip vehicle routing problems with time windows (MTVRPTW). In the presented model, the vehicles unload the cargo collected from customers in a warehouse that has a limited unloading capacity. A solution algorithm based on branch, price, and cut, for the proposed MTVRPTW model, is also proposed. Rezaei et al[1] have presented a multi-objective model for the routing problem of multiple vehicles (MO-MVRP) in critical conditions for blood supply to injured people. In this presented model, the arrival time of the vehicles and the amount of blood collected is considered the objective functions of the problem. CPLEX deterministic solution is used to solve the proposed model. In this study, Wang et al[22] introduce a Collaborative multi-depot vehicle routing problem with dynamic customer demands and time windows considering resource sharing and dynamic customer requirements (CMVRPDCDTW). For this purpose, a bi-objective optimization model is developed to optimize vehicle routes while minimizing the total operating cost and the number of vehicles. To solve the proposed model, a hybrid algorithm consisting of an improved k-medoids clustering algorithm and multi-objective particle swarm optimization is presented to find near-optimal solutions. Hasanpour et al[23] have formulated a multi-trip open vehicle routing problem (MTOVRP). For this purpose, an appropriate integer programming model has been developed to minimize the total costs of buyers. To solve the model, an algorithm based on decomposition is presented, which divides the problem into two parts. In the first stage, tactical decisions are made regarding supplier selection and type of cooperation. In the second step, the visit sequence of each vehicle is specified. Nozari et al[24] have presented a multi-depot Vehicle routing problem under uncertainty (MDVRP). The primary goal of the proposed model is to locate warehouses and production centers and route vehicles to distribute medical goods to hospitals. A robust fuzzy method with uncertain parameters such as demand, transmission, and distribution costs has been used to solve this model. The effect of uncertainty has been investigated using the Neutrosophic fuzzy programming method. Xiao et al[25] have proposed a multi-stage vehicle routing problem based on task grouping for the VRP-Energy constraint in disasters. To solve the problem, they have presented an innovative algorithm based on k-means clustering and a genetic algorithm. Ramirez et al[26] have presented a multi-trip routing problem model with a vehicle to transport blood units from collection sites to a blood center (MSVR-TG). This problem is modeled as a multi-trip vehicle routing problem with the increasing profit, for which a mixed integer linear programming is proposed. Kuo et al[27] designed a two-stage vehicle routing problem with time window (VRPTW) to solve the disruption. The first stage is the supply chain in ideal condition, while the second one is the supply chain in disrupted condition since the increase in the supply chain complexity also leads to more vulnerability to disruptions. Zahedi et al[28] developed a bi-objective mathematical model for the capacitated electric VRP with time windows and partial recharge (BMCVRPTW). The first objective deals with minimizing the route to reduce the costs related to vehicles, while the second objective minimizes the delay of arrival vehicles to depots based on the soft time window. Li et al[29] establishment a bi-objective optimization model to solve the cooperative distribution problem of a multi-center hybrid fleet by integrating reverse logistics under real-time road conditions (VRP-RL). In Tale 1 based on the studies mentioned above, in Table 1, the classification of relevant literature has been done.

Table 1. Literature categorized

Method	Logic		Period		Objective		Model	Year	Author	Sub Problem	Basic Problem
	Uncertain	Certain	Multi	Single	Multi	Single					
Meta heuristic	-	*	-	*	*	-	2E-LRP	2020	Yu et al	2E-LRP	LRP
Meta heuristic	-	*	*	-	*	-	MP-LRP	2021	Cheng et al		
Meta heuristic	-	*	*	-	-	*	MP-2E-LRP	2021	Wang et al		
Meta heuristic	-	*	-	*	-	*	2E-LRP	2021	Santos-Gandara et al		
Exact-Meta heuristic	-	*	-	*	*	-	MO-2E-LRP	2021	Fallah-Tafti et al		
Meta heuristic	-	*	-	*	-	[illegible]	2E-BLRP	2021	Cao et al		
Benders	*	-	-	*	-	*	2E-SMCLRP	2022	Mohammad et al		
Meta heuristic	-	-	-	*	*	-	2E-DVRP-PSSs	2022	Xiao et al		
Meta heuristic	-	-	-	*	*	-	MW-2E-JDLRP	2022	Do et al		
Exact-Meta heuristic	-	*	-	[illegible]	*	-	G-2ECOLRP	2022	Heidari et al		
Meta heuristic	-	*	[illegible]	*	-	*	MTVRPTW-SDLT	2020	Niera et al	VRP	
Price Branch Cut	-	*	[illegible]	-	-	*	MTVRPTW	2021	Huang et al		
Exact	-	[illegible]	-	*	*	-	MO-MVRP	2021	Rezaei et al		
Meta heuristic	-	[illegible]	[illegible]	[illegible]	*	-	CMVRPDCDTW	2022	Wang et al		
Meta heuristic	-	*	-	*	-	*	MTOVRP	2022	Hassanpour et al		
Neutrosophic	*	-	-	*	*	-	MDVRP	2022	Nozari et al		
Exact	-	*	-	*	-	*	MSVR-TG	2022	Ramirez et al		
Meta heuristic	-	*	-	*	*	-	VRPTW	2023	Kuo et al		
Meta heuristic	-	*	*	-	*	-	BMCVRPTW	2023	Zahedi et al		
Meta heuristic	-	-	-	-	*	-	VRP-RL	2023	Li et al		
Exact	[illegible]	*	*	-	*	-	2E-MPMORLHVP	2023	Current search		

Based on the reviewed studies and the classified table 1, according to the obtained knowledge, a two-level multi-period model is rarely considered in a study. For example, in the study of Yu et al[10] and Santos-Gandara et al [13], a single-period two-level model is considered. Also, in the study of Cheng et al[11] and Wang et al[12], a multi-period one-level model is presented. Therefore, in this research, a multi-period two-level model is presented to fill the gap. At one level of the proposed model, the location allocation for dispatching vehicles and at the other level the route for transport vehicles is determined.

Therefore, based compared with the previous studies mentioned above, this study has several contributions of new and innovative contributions to theory and practice, which are presented as follows.

(1) It proposes a two-level multi-period logistics network based on the periodic features of logistics facilities and customers to attain facility location decisions and vehicle routes instead of independent operations among several logistics facilities.

(2) It provides consideration of the heterogeneous transportation resource strategy in the proposed 2E-MPLRP problem to optimize facility location decisions and vehicle schedules, thus improving transportation efficiency and obtaining heterogeneous resource configuration among multiple logistics facilities in different periods.

(3) Presenting a bi-objective mathematical planning model considering different periods, heterogeneous transportation resources, and transportation requirements between locations to minimize the total cost of logistics operations and the duration of vehicle repairs, in case of failure in a two-level multi-period distribution network, has been taken into account.

3. Research Methodology

In this section, all the items, including variables, parameters, functions, and limitations of the mathematical model developed for mathematical modeling, which will be able to explain the problem model and its solution, and apply the facility failure time in the model, will be explained. In this paper, developed a routing and locating problem for 5 cities with different types of vehicles. This paper presents a mathematical model for the design of accurate locating and routing of vehicles in a supply chain. The presented model is a mixed integer linear programming model with two objective functions to minimize the amount of transportation cost and the duration of vehicle repair. The designed supply chain includes several cities, different vehicles, and transportation routes in which transportation starts from a certain point and continues with allocation to different cities and route determination. Thus, the dispatch of vehicles through different vehicles with the lowest cost and the shortest repair time in case of breakdown is done from the starting point. Using the epsilon constraint method, the multi-objective model is transformed into a single-period single-objective linear programming (LP) model. Due to the multi-period nature of the supply chain, all transportation costs and repair times are calculated for three periods. In Figure 2, the overall mechanism of the supply chain, and in Figure 3, the proposed model for allocating several cities and determining the route are shown.

Figure 2. Supply chain execution mechanism

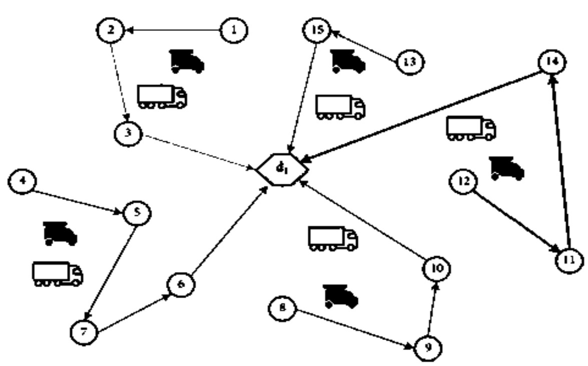

Figure 3. The overall mechanism of the considered model

For modelling there are some assumption follow as:

- We have different time periods.
- There are different types of vehicles.
- Special work service is available for each vehicle.
- Planning is done on two levels.

The process of implementing problem modeling in the current research has the following steps.

First step: building and developing a suitable mathematical model in accordance with the main characteristics and assumptions of the problem;

Second step: coding the model in a suitable software environment such as GAMS software;

Third step: sensitivity analysis;

If we do not get an answer from the mathematical model in the second step, we go back to the first step and after making changes, we redesign the model again. In Figure 4 research methodology flowchart is depicted.

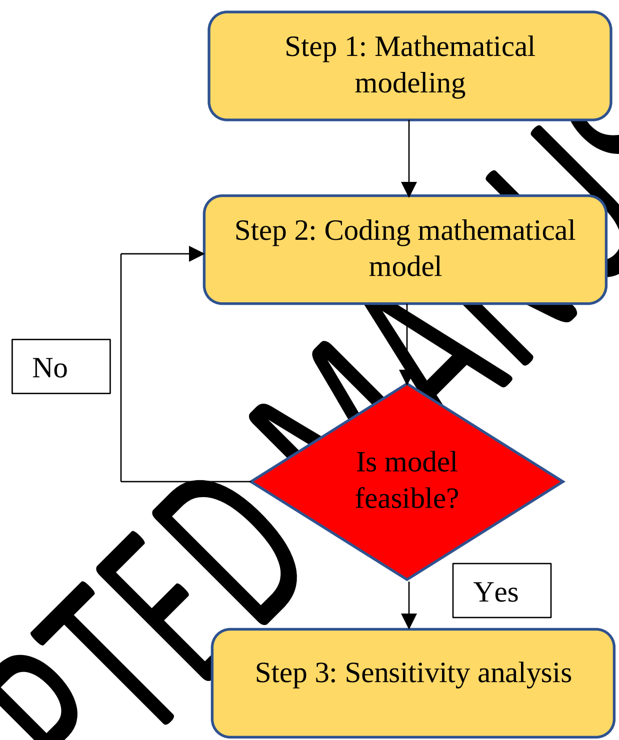

Figure 4. Research methodology flowchart

3.1 Objective functions

In this part of mathematical modeling, the components of the two-objective programming problem that has two objective functions are introduced. The objective functions of this research are: minimizing the amount of transportation cost in each route and minimizing the vehicle repair time during transportation. The details of these objective functions are shown in equations (1) and (2)

$$\min \sum_{t=1}^{T}\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{k=1}^{K}\sum_{n=1}^{N} c_{ijt}\,(x_{ijkt}+x_{ijnt}) + c'_{ijt}\,(x_{ijklt}+x_{ijnt}) \tag{1}$$

The first function guarantees that the cost of allocating route i to j with vehicle k and n in period t is minimized.

$$\min \sum_{t=1}^{T}\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{k=1}^{k}\sum_{n=1}^{N} w_{it}\,cr_{it}\,(x_{ijkt}+x_{ijnt}) \tag{2}$$

The second function guarantees that the repair time of each k and n vehicle that has a breakdown on the route i to j in period t is minimized. In this research, the epsilon constraint method has been used to solve the two-objective problem. Based on this method, one of the objective functions is considered a constraint by considering an upper bound. In this research, the second objective function $\sum_{t=1}^{T}\sum_{i=1}^{r}\sum_{j=0}^{s}\sum_{k=1}^{k}\sum_{n}^{N} w_{it} cr_{it} (x_{ijkt} + x_{ijnt})$ considering the upper limit of f_i as a limit is considered as $\sum_{t=1}^{T}\sum_{i=1}^{r}\sum_{j=0}^{s}\sum_{k=1}^{k}\sum_{n}^{N} w_{it} cr_{it} (x_{ijkt} + x_{ijnt}) \le f_i$. Therefore, this limit, by adding to the set of constraints, guarantees that the total repair time of each vehicle from the repair time of the vehicle at node i does not exceed the value of f_i.

3.1.1 Constraint

In this section of the research, all constraints are introduced according to equations 3-18.

$$\sum_{t=1}^{T}\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{k=1}^{K} x_{ijkt} \le k \tag{3}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{r}\sum_{j=1}^{s}\sum_{n=1}^{N} x_{ijnt} \le k \tag{4}$$

$$\sum_{t=1}^{T} x_{ijkt} = 1 \qquad i = 0; k = 1, \ldots, k \tag{5}$$

$$\sum_{t=1}^{T} x_{ijnt} = 1 \qquad i = 0; k = 1, \ldots, k \tag{6}$$

$$\sum_{t=1}^{T} x_{ijnt} \sum_{j=1}^{s} x_{ijkt} = 1 \qquad k = 1, \ldots, k \tag{7}$$

$$\sum_{t=1}^{T} \sum_{j=1}^{s} x_{ijnt} = 1 \qquad k = 1, \ldots, k \tag{8}$$

$$\sum_{t=1}^{T} \sum_{k=1}^{k}\sum_{j=0}^{s} x_{ijkt} = 1 \qquad j \ne i \tag{9}$$

$$\sum_{t=1}^{T} \sum_{n=1}^{N}\sum_{j=0}^{s} x_{ijnt} = 1 \qquad j \ne i \tag{10}$$

$$\sum_{t=1}^{T} \sum_{k=1}^{k}\sum_{i=0}^{r} x_{jikt} = 1 \qquad j \ne i \tag{11}$$

$$\sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=0}^{r} x_{jikt} = 1 \qquad j \neq i \tag{12}$$

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=0}^{N} x_{ijk}\,(t_{ij} + f_i + w_i) \leq r_{kt} \qquad j \neq i \tag{13}$$

$$\sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{j=0}^{N} x_{ijn}\,(t_{ij} + f_i + w_i) \leq r_{nt} \qquad j \neq i \tag{14}$$

$$T_0 = W_0 = f_0 = 0 \tag{15}$$

$$e_i \leq (T_i + w_i) \leq l_i \tag{16}$$

$$\sum_{t=1}^{T}\sum_{k=1}^{k}\sum_{i=0}^{r} x_{ijkt} - \sum_{t=1}^{T}\sum_{k=1}^{k}\sum_{i=0}^{r} x_{jikt} = 0 \tag{17}$$

$$\sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=0}^{r} x_{ijnt} - \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=0}^{r} x_{jint} = 0 \tag{18}$$

Equations 3 and 4 guarantees that the selected routes must not exceed the number of vehicles. For maximum coverage, the presence of this limit in the set of equations is vital. Otherwise, a part of the route will not be covered. Equations 5 and 6 ensure that the route for transporting vehicles with vehicles k and n starts from the zero nodes (origin). Also, equations 7 and 8 guarantees that there is a round-trip route between two nodes for each vehicle type k and n. In the set of equations 9 to 12, it is guaranteed that the selected paths i to j start from zero origins. They also guarantee that no paths are starting from any node and ending at the same node. Equations 13 and 14 ensure that the sum of the travel time between two nodes, the repair time of each vehicle at each node, and the waiting time for receiving service to perform repairs does not exceed the maximum time each vehicle is allowed to move on the route i to j. Constraint 15 guarantees that the total travel time between two nodes starting from the origin node, the vehicle repair time at the origin node, and the waiting time to receive the service to perform repairs at the origin node are equal to zero. By taking $t_{ij} + f_i = t_i$ into account, finally, constraint 16 guarantees that the duration of moving, repairing, and waiting time for receiving vehicle repair service does not exceed the earliest time and the latest time. Constraints 17 and 18 guarantee the balance. That is, if any vehicle enters any city in any period, the same vehicle will also leave that city. The above equations are applied to be used in 5 cities. As shown in Figure 5, the flow of vehicles is maintained through two-way routes between both cities. Considering the above set of equations, we want to determine the routes of vehicles, the time of arrival in each city, and the waiting time.

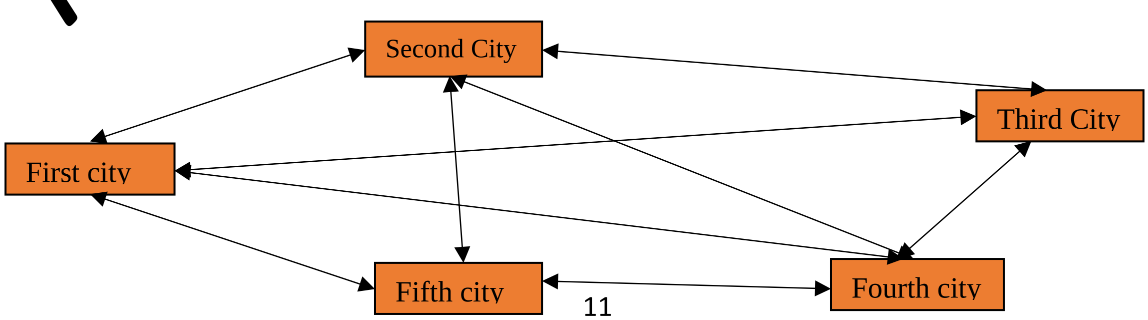

Figure 5. All possible routes for visiting

3.2 Solving approach

The solution method in this problem is based on the epsilon method of limitation. In the epsilon constraint method, one function is selected from among different objective functions and other objective functions are converted into constraints by considering an upper limit, and the problem is converted into a single objective linear programming model and is solved in the usual way of linear programming. One of the accurate methods of obtaining optimal Pareto solutions is the use of the epsilon constraint method, which was first presented by Al-Jadaan. The main merit of this method compared to other multi-objective optimization methods is its application to non-convex solution spaces, because methods such as a weighted combination of objectives lose their effectiveness in non-convex spaces. The computing time of an algorithm is one of the important features of any algorithm to evaluate it. Since one of the main weaknesses of the algorithms based on exact search, including the epsilon method, is the high limitation of their computing time, it is crystal clear that the use of the meta-heuristic algorithm triggers a sharp reduction in the computing time. One of the modified versions of the epsilon constraint method is the framework presented by Pirouz and Khorram and Abolghasmian et al[30]; and Abolghasemian et al[31]; Abolghasemian et al[32] has recommended its use owing to having two major advantages. One of the benefits of this method is to reduce the search space to find non-dominant points. Another advantage of this method is its less execution time compared to the original method. According to this method, we first solve the single objective optimization problem for each objective. Then we determine the length of the step. Then we generate the set of suitable points and finally solve the single objective optimization and estimate the Pareto frontier. In this method, we always optimize one of the objectives, provided that we define the highest acceptable limit for the other objectives in most of the limitations. For a two-objective problem, we will have the mathematical representation according to equation (19):

$$
\begin{aligned}
&\min f_1(x) \\
&s.t \\
&f_2(x) \le \varepsilon_2 \\
&x \in s
\end{aligned}
\tag{19}
$$

By changing the values on the right side of the new constraints of ε, the Pareto frontier of the problem will be obtained.

4. Findings

4.1. Case Study

Kale Dairy Products Company is one of the subsidiary companies of Suliko Group, which was established in 1991. During the past years, this company has expanded greatly in terms of production capacity, quality and variety of products, national distribution network and export. In such a way that it is currently considered as one of the two leading companies in the dairy products market in the whole country, which exports to the United States of America, Russia and Iraq. The supply chain of sending the products of Kale Company is very wide and diverse, which is done through different trucks for long distances and for the area within the province, from smaller vehicles to transport the product. In this research a strategy for transporting heterogeneous vehicles over periods of time for handling and distribution in Kale supply chain is presented. In this section results of implementing of proposed model in Kale supply chain is presented. For this purpose, the distribution of dairy products of Kale Company is considered in five cities of Mazandaran province, which will be done in three periods from the origin of Amol to the cities of Babol, Sari, Ghaemshahr, Neka and Behshahr. The data of this research has been collected based on the registered records that are available in Kale Company.

4.2. Practical results

In this section, the calculation results of the proposed model are shown. GAMS software has been used to run the model. The proposed model can determine the rout of visiting places for different vehicles. It also specifies the arrival time of each vehicle and the waiting time to receive the necessary repair service. In Table 2, the service route from each city to another adjacent city is shown by solving the problem through zero and one programming. In this table, for the calculation of the first objective function, the value of the displacement cost is considered as the objective function of the problem, and the second objective function is added as a limit in the constraints. It is obvious if the path is selected for the visit, the value of one is assigned to it, and otherwise, the value of zero.

Table 2. Determining the route to visit the place

The objective function (Currency)			x_{ij}	Destination	Status		
$t=3$	$t=2$	$t=1$			$t=3$	$t=2$	$t=1$
$f_1^* = 1675$	$f_1^* = 1$[illegible]88	$f_1^* = 1505$	First city	First city	0	0	0
				Second City	0	1	0
				Third City	1	1	0
				Fourth city	0	0	1
				Fifth city	0	0	1
			Second City	First city	1	0	0
				Second City	0	1	0
				Third City	1	0	1
				Fourth city	1	0	0
				Fifth city	0	1	0
			Third City	First city	0	0	1
				Second City	1	0	0
				Third City	1	0	0
				Fourth city	0	1	0
				Fifth city	1	0	0
			Fourth city	First city	1	1	1
				Second City	0	0	0
				Third City	0	0	0
				Fourth city	1	0	0
				Fifth city	0	0	0
			Fifth city	First city	0	0	0
				Second City	0	0	1
				Third City	0	1	0
				Fourth city	0	1	0
				Fifth city	1	0	0

As can be seen in Table 2, it is not possible to travel from any city to the same city. Also, allocating and determining the route in the first period has the lowest cost. Thereby, the allocation in the first period is the most optimal possible state among the periods considered. Furthermore, it is possible to travel to cities 4 and 5 only through city 1. Vehicles from city 2 only travel to city 3 and cover it completely. Part of the demand in city 1 goes through city 3 and the other part goes through city 4. Eventually, there is only traffic from city 5 to city 2. In Table 3, the cities of vehicle dispatch and the visited cities are shown. Also, in Figure 6, the optimal route for locating and routing vehicles is shown under the condition that the first objective function is considered as the objective of the problem in the epsilon constraint method.

Table 3. Details of cities' dispatch and visit

Vehicle type	Visited City	City of vehicle dispatch
k and n	The fourth and fifth city	First city
n	Third City	Second City
n	First city	Third City
n	First city	Fourth city
n	Second City	Fifth city

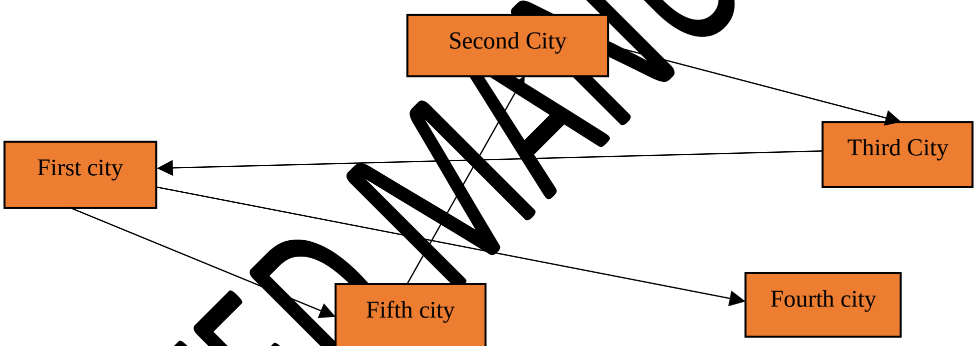

Figure 6. The optimal route for vehicles to visit cities

Also, in Table 4, for the calculation, the second objective function, i.e., the value of vehicle repair time, is considered as the objective function of the problem, and the first objective function is added as a limit in the constraints. It is palpable, if the vehicle is on a route that needs to be serviced to continue the repair operation, the value of one is assigned to it and otherwise, the value of zero.

Table 4. Allocation of service workers to continue the operation

The objective function (Minute)			x_{ij}	Destination	Status		
$t = 3$	$t = 2$	$t = 1$			$t = 3$	$t = 2$	$t = 1$
$f_2^* = 57$	$f_2^* = 50$	$f_2^* = 55$	First city	First city	1	0	1
				Second City	1	0	1
				Third City	1	1	1
				Fourth city	0	1	0
				Fifth city	0	1	0
			Second City	First city	1	0	0
				Second City	1	0	0

				Third City	1	1	1
				Fourth city	1	1	0
				Fifth city	1	0	1
			Third City	First city	0	1	1
				Second City	0	0	1
				Third City	0	0	0
				Fourth city	0	1	0
				Fifth city	1	1	0
			Fourth city	First city	1	1	0
				Second City	0	1	0
				Third City	0	0	1
				Fourth city	0	0	1
				Fifth city	0	0	1
			Fifth city	First city	0	0	0
				Second City	1	1	0
				Third City	1	1	0
				Fourth city	0	0	1
				Fifth city	0	0	1

As can be seen in the results of Table 4, the status of announcing the need for work service during different periods is shown. Based on the value obtained for t=2, has the lowest repair cost. Therefore, the results are described according to it. The objective functions in the period t=2 to perform repair operations are as follows:

- If the vehicles in the 1st city need to be repaired, the serviceman should go from the 3rd, 4th, and 5th cities.
- If the vehicles in the 2nd city need servicing, the serviceman should go from the 3rd and 4th cities.
- If the vehicle needs service in the 3rd city, the serviceman should go from the 1st, 4th and 5th cities.
- If the vehicle needs service in city 4, the serviceman should go from the first and second cities.
- In case the vehicles in city 5 need service, the serviceman should go from the second and third cities.

In Table 5, the cities of vehicle dispatch and the visited cities are shown. The results are shown based on the second objective function.

Table 5. Details of service worker dispatch and visit

City of dispatch	Visited City
First city	The third and fourth city
Second City	The fourth and fifth city
Third City	First, the second and fifth city
Fourth city	First, the second and third city
Fifth city	First and third city

The optimal time to enter each city and the waiting time for vehicle repair are also shown in Table 6. Based on the calculated time advance, it is determined that the maximum possible time to enter each city and repair vehicles is 19 minutes. In Table 6, the time related to the arrival of the vehicles and the duration of the repair of each of them, if needed and advanced, are shown.

Table 6. Optimum time of arrival and repairs of vehicles

City	Arrival time	Repair time	Advancing time
First city	0	0	0

Second City	3	16	19
Third City	6	13	19
Fourth city	7	7	14
Fifth city	7	7	14

Based on the calculated time advance, vehicles going to cities 3 and 2 will leave the desired after receiving the repair service city at time 19, in addition to being visited. The same occurs for vehicles going to cities 4 and 5 within 14 minutes after the start of the process. In addition, by sorting the arrival time in each of the visited cities in ascending order, we can determine the priority order of repairs. According to the specified priority, first, the process starts in the first city, then the third and second cities, and finally, the repairs are completed by referring to the fourth and fifth cities.

4.3 sensitivity analysis

In this section, the sensitivity of the values of ε will be measured on the value of the objective functions, and the results, including reliable values by determining the distance of ε for the objective functions will be reported. To do that, different values for ε are defined in Table 7 and the objective functions are solved using them. As can be seen in Table 7, the values of the objective function do not show a significant change with the increase of ε up to a certain value, but from some point on (for example, the second objective function), the increase in the value of ε reports a remarkable increase in the objective function values (epsilon change from 600 to 900). Based on the obtained results, for testing different values of epsilon, the feasible region and the improving vector of the objective functions have been created. According to the results, the level of significant changes of epsilon between 50 and 900 has been determined as the improvement operator. Determining this interval specifies that if the epsilon value is considered to be less than 50 and greater than 900, the answer to the problem is outside the feasible area. Therefore, the range of epsilon variations to search for the local optimal solution for the first objective function is 650 as the optimal solution for the first objective function occurs on this point. The optimal situation for the second objective function is obtained at epsilon 600. Hence, if epsilon is selected between 600 and 650, non-dominant answers are attained for the problem, otherwise, non-dominant answers are considered. Table 7 shows the results of solving the model with a step length equal to 50. Using the values obtained from calculating the value of the objective function through different epsilons, the Pareto frontier created for the problem is shown in Figure 7.

Table 7. Model solution results with ε-constraint method

Repair time (minute)	Cost (currency)	ε
57	1512	50
55	1563	100
62	1582	150
57	1550	200
68	1530	250
75	1513	300
58	1598	350
54	1565	400
56	1548	450
58	1507	500
52	1580	550
50	1512	600
72	1505	650
98	1550	700

78	1560	750
81	1513	800
82	1515	850
70	1598	900

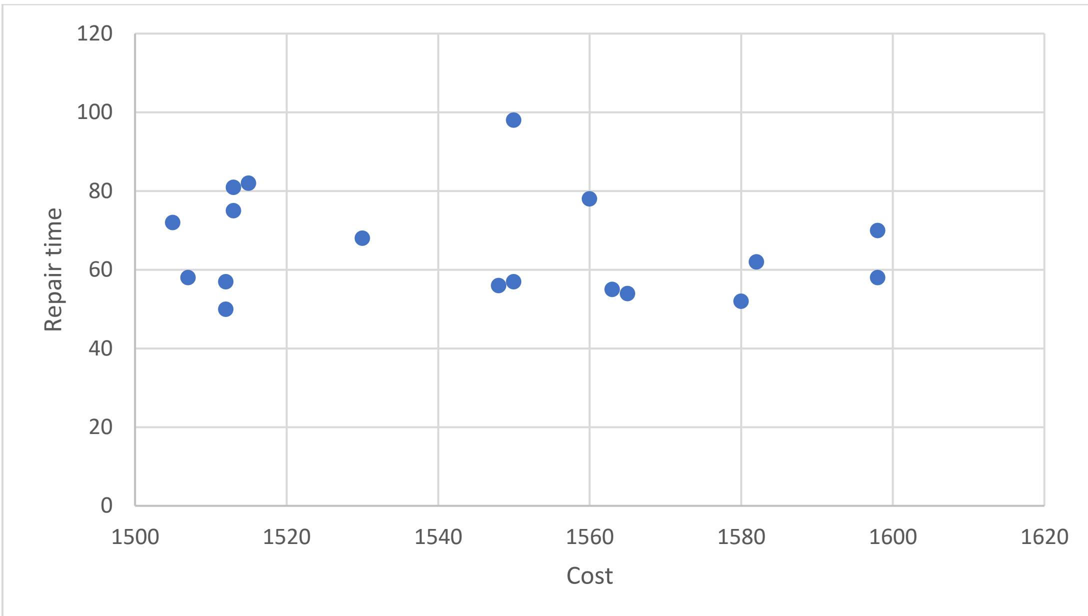

Figure 7. Pareto frontier

4.4 Managerial Insight

Current research can provide valuable managerial insight in several ways:

1. **Optimal resource allocation:** The research can help managers understand and optimize the allocation of heterogeneous vehicles and other resources across different time periods. It provides insights into how to assign vehicles efficiently to meet multiple objectives, such as minimizing transportation costs, maximizing customer satisfaction, or minimizing carbon emissions.
2. **Facility breakdown management:** By incorporating facility breakdowns into the model, the research offers insights into how managers can effectively handle unexpected disruptions in operations. It provides guidance on how to reconfigure the routing and locating of vehicles to minimize the impact of breakdowns on overall system performance.
3. **Trade-off analysis:** The model's multi-objective nature enables managers to analyze trade-offs between different objectives, such as cost and repair time. By quantifying these trade-offs, managers can make more informed decisions and identify optimal solutions that balance conflicting goals.
4. **Sensitivity analysis:** The research can provide insights into the sensitivity of the model's results to various parameters and assumptions. Managers can use this information to identify critical factors and assess the robustness of their decisions.

Overall, the research offers managers a comprehensive decision support tool that can guide them in making informed decisions regarding multi-period, multi-objective routing and locating of heterogeneous vehicles, taking into account facility breakdowns. It provides valuable insights into optimizing resources, managing disruptions, and finding the best trade-offs to achieve operational efficiency and customer satisfaction.

5. Conclusion

This study proposes a multi-objective optimization model for optimal route allocation of vehicles to visit cities in a two-level supply chain with heterogeneous vehicles. Two objective functions are considered: minimizing the total cost and duration of vehicle repair. To solve the proposed mathematical model, the epsilon constraint method has been used. This method can determine the dominant and non-dominant answers without the subjective judgments of experts. The application of proposed model has been applied through the numerical solution of the location of the optimal route allocation for the visit of different vehicles, which confirms the applicability of the mathematical model. The numerical test results, in terms of optimal route allocation by solving the proposed model, clearly determine the current route allocation plan for the best routes for each vehicle to visit each city. Current study may have several limitations, including proposed model is based on certain assumptions that might not reflect the real-world conditions accurately. These assumptions can limit the applicability and generalizability of the model. Also, Mathematical models often simplify complex real-world scenarios to make them mathematically solvable. These simplifications may lead to oversimplification and neglect certain important factors, leading to limitations in the model's accuracy and practicality. The model's accuracy and effectiveness strongly depend on the availability and quality of data used for parameter estimation and validation. Limited or inaccurate data can significantly impact the reliability and usefulness of the model's results. The proposed model may be specific to certain contexts, such as certain types of transportation networks or vehicle fleets. The model's applicability and generalizability to different scenarios or industries might be limited. Therefore, it is important to consider these limitations while interpreting and utilizing the findings of the research. Researchers should strive to address these limitations and explore further avenues for improvement and validation. Decisions about facility routes in response to site visits are usually made based on the experience of decision-makers or temporary decision-making. Hence, the proposed model improves both efficiency and effectiveness in this field, and the findings of this study provide guidance for improvement of decision-making for route allocation in the context of locating-routing and will be beneficial for designing a suitable strategy in the future. For further research, it is suggested to add the property of the time window for vehicle departure in the proposed model. Also, due to the fact that the demand is not known in the real world, a robust planning model should be provided by considering the random feature for this parameter.

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