Cost Optimization of Projects with Fuzzy Duration Activities Using Genetic Algorithms

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Abstract— The minimum cost is a crucial target of almost all types of construction projects, and it is achieved by an efficient scheduling. However, each project is unique and duration of activities involved in a project often cannot accurately be predicted. In this research, fuzzy sets were the solution. One prominent point of this research was considering the level of risk acceptance, based on which, crisp durations for activities were attained. In other words, fuzzy scheduling was turned into crisp scheduling. Then, a method based on genetic algorithm was selected to select the operating mode for the smallest project total costs. The last stage of the proposed method was determination of the fuzzy project cost. Simplifications made in this study make it possible to find optimum solutions in complex problems. Next, an example of a construction project was used which substantiated that genetic algorithm with its selected input data (population and generation number) and criterion of selecting surviving chromosomes for next generation (roulette wheel principle) could deliver reliable outcomes and provides a tool for handling real-world construction projects. Furthermore, the performed sensitivity analysis proved that the proposed model is not much sensitive to large variation in the values of acceptance level of risk. Finally, for the purpose of validation of the effectiveness of the proposed model, the case study was solved by three widely-used approaches. This comparison (at least 17% improvement in solutions) was a reason for the fact that the presented model is a tool helping project managers a lot.

Keywords: Execution Modes, Fuzzy sets, Genetic Algorithms, Project scheduling.

1. Introduction

It is well-established that it is difficult to predict the duration of an activity involved in the project closely. This is because small information is available at the early stage of the project. Fuzzy sets are used to represent the imprecise activities duration. Many researchers have done many researches in this area, for different goals. For the purpose of the project critical path, Zammori, Braglia, Frosolini, integrate fuzzy logic and multi-criteria decision-making methods (MCDM) to show the critical path network with fuzzy duration [1]. Dorfeshan et all proposed a simple approach to solve critical path problem in project network by a new enhanced multi-objective Optimization of Simple Ratio Analysis Approach with Interval Type-2 Fuzzy Sets [2]. David A. Wood and Haji Yakhchali and proposed an approach to analyze the critical path for a project network with activity becoming fuzzy number [3, 4]. Chen, Huang, provided a new interval-valued fuzzy multi-objective approach for project time—cost—quality trade-off problem with activity crashing and overlapping under uncertainty [5]. For the purpose of project risks analysis, Saptarshi Mandal et all used fuzzy similarity value and possibility theory based approach [6]. Mengqi Zhao et all proposed a Construction Schedule Robustness Measure Based on Improved Prospect Theory and the Copula-CRITIC Method [7].

For the purpose of the project duration, Ali Moradi Vartouni et all considered a multi-mode resource constrained project scheduling problem applying a hybrid genetic algorithm and fuzzy set [8]. For the purpose of project time and cost, Ashish Sharma provided a Fuzzy Multi-Objective Genetic Algorithm Based Resource Constrained Time-Cost Trade-Off Model under Uncertain Environment [9]. He and Diane analyzed effect of reward - fine structure in project payment plan in two different searches [10, 11]. Sakellaropoulos and Chassiakos developed a solution method considering direct cost, indirect cost and bonus/penalty [12]. Wisitsak Tabyang and Vacharapoom Benjaoran proposed a Modified Finance-Based Scheduling Model with Variable Contractor-to-Subcontractor Payment Arrangement [13]. Duc Hoc Tran incorporated multiple-objective social group optimization and multi-criteria decision-making methods in Time-cost analysis [14].

None of the mentioned research mimicked the planning problem in generalized construction projects and converged the true search space with a good solution and coverage. Not to mention that the applied methods are used under the hypotheses that are seldom satisfied. There is a need for a reliable and robust approach which can be used in all stages of the construction

process. There is no need to argue that the dynamic nature of construction activities may require project rescheduling within weeks or even days. As a result, to determine the most appropriate schedule for the construction process during its lifecycle, a great number of model running is required. Thus, the reduction of the computational time should be enough satisfied in using an approach. In the present study, this approach is a type of meta-heuristic optimization whose capabilities were extended and one of the main practical limitations of it was resolved.

In addition, Cost or time minimization was the aim of most implemented researches about fuzzy scheduling. In the other words, the desired completion project time at a minimum cost or the reverse was focused. But time and cost are dependent on each other. If allocation more resources decrease the project time, direct project cost would be increased, indirect project cost would be decreased, late penalty/early bonus would also be changed. A sophisticated project manager knows that projects include many activities linked in a complex way and can be done by using alternative modes which define the subset of the resources needed and its uncertain duration. Before preparing an appropriate plan for whole project lifecycle, he/she tries to augment his knowledge on the project as a whole and carefully analyze each task of the project, and finally extracts numerous combinations of project time and cost resulted from his/her different decisions. At the same time, it is necessary to complete a project as soon as possible and before an allowed deadline, and it seems desirable to develop a method that allows to incur a high penalty (because of project delay) and bonuses (because of project acceleration) on the evaluation function, instead of using direct project costs and indirect project costs as the sole deciding factors.

Although, literature in the subject matter is extensive, none of the the previously mentioned studies integrated explicit consideration of multiple-execution modes, adequate consideration of uncertainty in projects, powerfulness, efficiency, effectiveness and addressing of importance of project completion time. The present simple procedure which neglect no crucial matter and takes into account all necessary issues such as extension to the fuzzy cases, surpasses many researches done in this area. It facilitates and increase the decision quality and its potentials and capabilities prove its originality and value.

The paper was organized as follows. Section 2 describes the fuzzy set theory and concepts of activity duration and the agreement index. The problem statement, transforming fuzzy activity times to crisp ones, the objective function (total project cost), schema of the genetic algorithm and the method based on genetic algorithms to solve the problem is presented in section 3. In section 4, an example was shown. Finally, Section 5 concludes the paper.

2. Fuzzy Sets and Membership Functions and their Agreement Index Concept

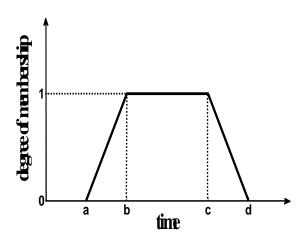
2.1. Fuzzy Sets and Membership Functions

Let X be a set of objects and x an element of X. Classical set A, $A \subseteq X$, as an element or set of objects $x \in X$ defined so that each x can either belong to the set A or does not belong. By defining a characteristic function for each element of x in X, a set of ordered pairs (x, 0) or (x, 1) can present a classical set A, which indicates $x \notin A$ or $x \in A$, respectively. Fuzzy set expresses the degree to which an element belongs to a set and therefore, the function of a fuzzy set is allowed to take values between 0 and 1 which represents the degree of membership of an element in a given set. This characterized function is called a membership function for fuzzy set A, which is defined as: If X is a set of objects commonly denoted by x, then a fuzzy set A in X can be defined as a set of ordered pairs:

A= $\{(x, \mu_A(x))|x \in X\}$, where $\mu_A(x)$ is called the membership function. Memberships function relates each element of X to a degree of membership (or membership value) between 0 and 1 [15].

For example, a trapezoidal membership function $\tilde{T} = (a,b,c,d)$ can describe the uncertain duration of an activity (Figure

1). This membership function $\mu_d(x)$ represents the degree of the probability that X will realize the activity duration. Values in the range [b, c] are the most recent period of activity and values in [a, b] and [C, D], which has a little chance to be realize as duration of activity.



The fuzzy theory addresses situations where application of classical mathematics is unable to model human concepts and thoughts and they are expressed by imprecision and vagueness. These situations are plentiful on real-life construction projects such as when an expert present duration of activities due to the dependencies induced by nature of these projects. There is doubt and uncertainty regarding his/her judgment of durations and his/her exact concepts and correct statements and judgments are expressed with fuzzy numbers.

2.2. Agreement Index Concept

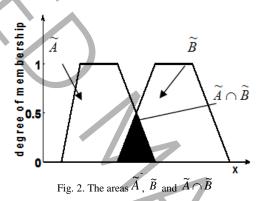
The agreement index AI (A, B) is used to measure the possibility between two fuzzy events by measuring the two fuzzy events agreement. AI is used in most of management procedures (e.g. Criticality measurement in fuzzy project scheduling by Mohammad A. Ammar and Sherif I. Abd-ElKhalek [16]; resource-constrained project scheduling with fuzzy activity durations by Yisong Yuan at all [17]).

 $AI(A, B) = Area(A \cap B)/Area(A),$

Where Area (A) =
$$\int \mu_A(t)dt$$
 and Area (A \cap B) = $\int \mu_{A \cap B}(t)dt$

Figure 2 represents the areas \widetilde{A} , \widetilde{B} and $\widetilde{A} \cap \widetilde{B}$.

Agreement Index (AI) represents the percentage of a fuzzy event (A) is located within the boundaries of a fuzzy event (B). It also represents the satisfaction level of fuzzy event (A) when the event fuzzy (A) is compared with other fuzzy event (B). AI is always less than 1 (100%), and the event A is fully consistent to event B when AI (A + B) = 1. From a practical viewpoint, the agreement index is a good indicator because it adequately considers the shape of fuzzy events.



3. General Algorithm for Selecting the Execution Mode for Real Activities with Fuzzy Duration in Minimizing Project Cost

3.1. Problem statement

Time and cost are two major concerns in construction projects. In the construction industry, contractors are usually to estimate project duration and cost of a new project making use of previous experience. Typically, a project will be broken down into activities to which resources can be assigned and their durations and cost are estimated. The activities with respect to work sequences are linked to figure of a network. Critical path method is used to analysis the network to show project duration. Overall, greater sources devoted to activities, the lesser time and more cost will be required to finish the project (work duration is a discrete function of sources). In other words, any activities can be done by using replaced states which define the subset of required resources. This Interrupted time-source relationship results in discrete time-cost relationship. A conventional Interrupted time-sources curve is shown in Figure 3.

This trade-off between time and cost gives construction planners both challenging opportunities to work out the best construction plan that optimize time and cost to complete a project. In many cases, projects are required to be completed within certain duration. Adjustment is needed to change the execution mode for each activity that yields the desired duration at a least cost. The afore-stated cost considers only costs associated with each activity execution and the corresponding total project cost is stated as its direct cost. In a project, however, other cost types typically incur.

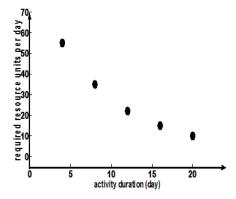


Fig. 3. A typical activity time-resource curve

Indirect project costs include general expenses that cannot directly be attributed to particular activities and exist regardless of activity progress (e.g. General office expenses). Indirect costs are typically assumed to be proportional to project duration. In addition, penalties for late project completion or bonuses for early project delivery apply in most cases.

In addition to afore-stated problems, projects are subject to much uncertainty that lead to uncertainties in the activities durations. In the other words, Activities are performed in one of different modes with a corresponding fuzzy duration. Hence, tools and methods need to be developed to aid construction planners in selecting the execution mode for each activity that optimize total project cost under uncertainty.

As mentioned before, some activities composing a project and each activity imposes some costs. These costs are categorized into two groups: independent of activity duration (e.g. building materials), dependent of activity duration (e.g. guard). Usually, the exercised productivity level complies with the minimum productivity requirements. But, it is apparent that productivity maximization in different ways such as providing efficient working condition leads to activity costs minimization because of minimization of the second class costs.

3.2. Transforming fuzzy activity times to crisp ones

As mentioned before the activity durations in project networks are usually difficult to estimate or find exactly, so it is reasonable to represent them as linguistic variables or fuzzy numbers. In this paper, the operation time for each activity in the project network is characterized as a positive trapezoidal fuzzy number. To deal with this problem, one approach was used to transform the fuzzy numbers to crisp ones and so fuzzy scheduling turn to crisp scheduling. The acceptable risk level of managers (α) was used and agreement index to carry out the transforming. The fuzzy activity time is represented by $\widetilde{T} = (a, b, c, d)$. To calculate the grade of possibility that the activity duration exceed a deterministic value (t), agreement index was used

The deterministic value (t) is transformed to fuzzy number $(\tilde{t} = (t, t, \infty, \infty))$ and $AI(\tilde{T}, \tilde{t})$ is determined. So, $AI(\tilde{T}, \tilde{t})$ is the grade of possibility that the activity duration exceed a deterministic value(t). In the other words, it defines the activity time risk. Figure 4. Represents \tilde{t} , \tilde{T} and $AI(\tilde{T}, \tilde{t})$.

3.3. Objective Function

In project scheduling, as project duration decreases, its direct cost increases but the indirect cost reduces. Hence, the problem is to find the project duration for which the total project cost is minimized. Therefore, the objective function

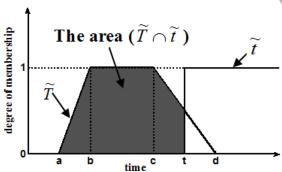


Fig 4. The activity duration and the grade of possibility that the activity duration exceed a deterministic value (t)

of the model is given by:

Min
$$\left[\sum_{i \in A} \sum_{k \in K(i)} c_{ik} x_{ik} + c_0 T + Y_{\perp}\right]$$

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Calculating Y: 
{
 If T>D_R
 Y=c_1 (T-D_R)
 Else
 Y=c_2 (T-D_R)
 End
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Where i is the activity indicator, A, the group of project activities; k, the indicator of a specific activity time-cost combination; K(i), the set of all possible time-cost combinations of activity i; c_{ik} , the cost of activity i corresponding to the time-cost combination k; x_{ik} equals to 1 if time-cost combination k is selected for activity i or 0 otherwise; T; the project duration; c_0 ; the project indirect cost per time unit; D_R the project deadline; c_1 ; the penalty rate per time unit of delay; and c_2 is the bonus rate per time unit. The first term of the objective function represents the project direct cost; the second one reflects the indirect cost and the third one reflect the penalties or bonuses for late or early project completion. The constraint of the model is given by:

$$\sum_{k \in K(i)} x_{ik} = 1, \forall i \in A$$

3.4. Basic schema of the genetic algorithm

There are two types of optimization methods: exact and inexact. Exact methods lead to optimal solutions, but they impose a high computational cost and for large-sized problems the exact calculus of optimal solution is cumbersome and frequently unfeasible. Since the problem of this study is very large (number of construction activities and their execution modes is great which lead to very large-scale optimization problems), gaining optimum solution is neither achievable and nor is feasible. To profit by the ability of inexact methods to handle and facilitate construction problems easily, meta-heuristics which require less computational effort were selected. The choice of these techniques is motivated by the need to be satisfied with near-optimum solutions. Evolutionary algorithms which are the most influential meta-heuristics provides this demand. Among them, genetic algorithm has its own strengths with reference to different problem space, because it has a more powerful and efficient search ability than its competitors.

Genetic algorithm (GA) was designed which was motivated by genetic evolution as general search strategy and optimization method (Goldberg, 1989). The main components of a genetic algorithm consist of a finite population of solutions, a chromosomal representation, a fitness function, and genetic operators. The chromosomal representation is an encoding that specifies a member of the population of solutions. Each solution in the population is encoded as a string of symbols (genes) called a chromosome. The fitness function is used to test a member in the solutions population. The members with the higher fitness values are assigned higher chance of choice for survival and reproduction. In each generation, the algorithm creates an entirely new population of people by selecting from the earlier population then mating to produce the new offspring for the new population. This process continues until the stopping criteria is met. The last chromosomes could usually represent the greatest or sub-optimal solutions to the problem [18]. Genetic operators meet to the reproduction: crossover and mutation.

In contrast to other local search algorithms that are based on manipulating one possible solution, a genetic algorithm considers a set of random solutions called population. Working with population permits us to find and explore properties that have common good solutions [19]. There have been several studies that apply genetic algorithms for solving project scheduling problem [20-24]. In this research, a GA-based analysis was used to decide which execution mode should be selected for each activity to decrease project cost.

3.5. Proposed GA Technique for Solving Resource Scheduling Problem

As mentioned before, based on level of risk acceptance, crisp durations are attained for activities. Hence, fuzzy scheduling is turned into crisp scheduling, which should be optimized by genetic algorithm. The genetic algorithm for solving the research problem is initialized with the first population of chromosomes generated randomly. Each gene in the chromosome string corresponds to an activity. There are as many genes in the chromosome as there are activities. The activities sequence in the chromosome corresponds to the sequence of the activities in the project sorted by their activity ID numbers in the network. The gene content corresponds to the execution mode of its corresponding activity. An example of a chromosome using this representation is shown in figure 5:

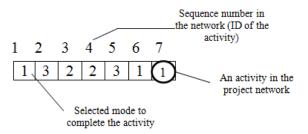


Fig 5. Description of chromosome

The number in the place i of the list represented the execution mode number of activity i. For example, the execution mode number of activity 3 is 2.

Three operations were done to produce a next generation with better chromosomes. The chromosomes with higher fitness values have more chance to be copied into the next generation. This can be done by randomly selecting and duplicating a chromosome with a chance that is proportional to the fitness value of the chromosomes. Since our goal is to produce the schedule with least cost, the objective function defined in section, is used as the scheduling performance measure.

After the reproduction operation, the crossover operator is applied to introduce new chromosomes by combining people in the new population with a chance called crossover rate. After the crossover operator is performed, the mutation operator is applied to each chromosome with a probability called mutation rate to introduce more variation in the current population.

The types of crossover and mutation operators used in this model are one-point crossover and uniform mutation. Crossover operates on two chromosomes at a time and generates the offspring by combining both chromosomes' features. The one-point crossover chooses a random cut-point and generates the offspring by exchanging the right parts of the two parent chromosomes (Figure 6). In this research, the right parts or the left parts of the two parent chromosomes are exchanged by random.

Besides crossover, mutation is a background operator which produces spontaneous random change in various chromosomes. Uniform mutation alters one gene in a chromosome, depending on the defined mutation rate [25, 26] (Figure 6).

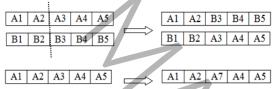


Fig 6. One-point crossover and uniform mutation

In each chromosome, the execution mode for each activity is decided and the corresponding activity duration and resource requirements are determined. Afterwards, the execution mode information will be input to the scheduling system, which will produce a possible schedule based on the project network (it is introduced at first). As a result, the total project duration, indirect project cost, penalties/ bonuses and finally the fitness value of each chromosome are calculated. The surviving chromosomes for the next generation are selected according to the roulette wheel principle. This means that the

choice possibility for a chromosome i is proportional to the ratio of $1f \frac{1}{f_i} / \sum_{i=1}^{pop-size} \frac{1}{f_i}$, where f_i is the fitness value of

chromosome i. Note that the fitness value is to be minimized here, i.e., small fitness values correspond to high selection probabilities. In fact, the mechanism operation of genetic algorithm was slightly modified from the original algorithm to increase its degree of credibility and surpass state-of-the-art algorithms like particle swarm optimization.

Usually, Project scheduling is a troublesome duty due to multiple-execution modes and precedence constraints. But, an innovative framework which is easy of computerization and understanding can determine the best schedule for project implementation by relying on small amount of data. Usually, in the field of construction management and execution, the project managers aim is to operate within some thresholds that maximize or minimize some levels that also allows for a compromise by which some issues can be fulfilled. This fully addressing some aspects leads to added computational processing which results in the fact that large amount of data should be available. This, itself results in high complexity of the used model and needs huge computational resources. As a result, it is noteworthy that in the implementation process of most of the literature on project scheduling method, even under the simplifying assumptions, complex resource types and diverse resource attributes are involved.

As a result, the applied model's complexity reduction is a determinant factor and requirement in complex projects such as construction ones. The number of parameters used in this study is small and the required knowledge is not much. The most challenging and data-demanding parameter is activities' duration which was simplified by means of some tools and techniques. Therefore, practical implementation of the proposed model which is an advanced computational optimization method is not time-consuming and the model is implemented on a computer system with a user-friendly interface which does not require considerable computational effort. With the help of novelties of the proposed model, one of enough expertise in scheduling and programming can correctly and easily extract some knowledge and consider and define some issues in real-

world situations to simulate scheduling. All this effort is made without much expense in terms of collection and computation time which is of immense importance to both practitioners and researchers.

During the course of a project, there is the possibility of the occurrence of interrupting events such as unexpected adverse environmental conditions, and it should be taken into account to ensure successful project completion. In fact, delay analysis should be done to deal with disruptions during the stage of project execution. In this way, the gap between practical needs and theoretical tools concerning project network methods will be closed. Because, in the field of scheduling, two robustness approaches are considered: quality robustness and solution robustness. Regarding quality robustness, the robust multimode discrete time/cost tradeoff problem is considered and solved by some algorithms. About solution robustness, various approaches are developed to cope with some disruptions. Activity duration disruptions and stochastic activity durations, are examples. Instead of addressing quality robustness or solution robustness separately, this study has concentrated on both of these two types of robustness. This is because of the mentioned easiness of development of the proposed model. This causes the wide-range applicability of the proposed model to actual projects which provides project managers with additional degrees of freedom to reiterate the application of the model during project due to some issues such as human errors and managerial inefficiencies.

In most practical cases just a little knowledge is available concerning each task of the project which causes variability of many issues such as starting time. For example, foreseeable and unforeseeable risks which causes unforeseen major design revisions require frequent changes in resource requirement. As we all know, proactive scheduling has been the subject of many research efforts that aim to generate robust baseline schedules that are protected against schedule disruptions. The more robust the baseline schedules are, the lower the adjustment costs will be during project execution. But, it is unavoidable that given the high degree of uncertainty in project environment, there are many decision making situations for which a project manager should find the optimal alternative from a number of feasible alternatives. It means that he/she needs to revise the initial developed model which was programmed to handle dynamic, real-time changes. The theoretical and practical issues of the presented method make this revision possible. This model's ability which increases its practical relevance, is the first major practical contribution of the present research. This decrease incurred costs when project managers adjust the starting times of the activities to deal with uncertainties which may be so powerful that change the project scope.

Emerging risks cause changes, delays or accelerations in one task which influence the overall project timeline and costs, because activities are often interdependent, and these precedence relations among project activities are usually influenced by simplifying assumptions in modeling scheduling problems. But, needless to say, activities are different with respect to their type of dependencies. The precedence relations between activities are usually governed by some factors, such as construction methods, safety consideration, space constraints, resource availability, project logic and traffic management and are of hard or soft character. If a relation connected with two activities is hard, its logical sequence cannot be changed, soft logics are those relations that in certain circumstances allow the activities connected by them to be scheduled as per a variety of work sequences or performed simultaneously. For example, it is physically possible to perform a repetitive activity in some units in the sequence instead of one unit, and weaken this predefined relation to generate other optional sequences. But, this changing the work sequence may result in longer activity durations and additional costs, and the change of project time and cost between which there is a trade-off, follow these changes. As mentioned before, in the present study, these two goals are merged into one objective. As a result, before and during project execution, it is possible and effortless to examine the impacts and variability of some precedence relationships.

4. Illustrated Examples and Computational Experiments

A project was considered to illustrate the algorithm application. This consists of seven activities. Each activity has three options to be selected from. The activity data are shown in Table I. Activity planning requires that all project activities start as soon as possible. The indirect project cost is 100 units per day. Further, a penalty at a rate of 250 units per day of delay applies after the 60th day while a bonus of 300 units per day is given for project completion before the 60th day. Resources cost rates are 20 and 15 units per day. The objective of the proposed model is to identify all solutions based on different values of α .

4.1. The first Experimental setup

The procedure was used with GA-related input data as follows: population size = 50 and generation number=1000.

Table 1. The activity data

Activity	Prec	Mode		Ti	me	•	Res1 requirement	Res2 Requirement
1	-	1	10	15	25	30	5	2
		2	8	12	15	20	7	4
		3	5	8	10	12	10	8
2	1	1	20	25	35	40	10	3
		2	15	20	22	28	15	7
		3	10	15	18	20	25	15
3	1	1	15	20	22	28	8	5
		2	10	12	15	18	15	9
		3	8	10	12	15	25	20
4	1	1	15	20	25	30	10	7
		2	12	14	18	20	18	15
		3	10	12	14	16	30	25
5	2	1	10	12	13	15	10	15

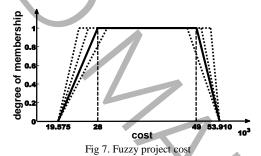
		2	8	9	10	12	15	28
		3	5	6	8	10	25	40
6	3,4	1	15	16	20	25	18	20
		2	14	15	18	20	30	30
		3	12	13	14	15	40	45
7	5,6	1	10	15	18	20	10	14
		2	7	10	12	15	20	18
		3	5	7	8	9	25	35

Crossover rate = 0.2, mutation rate = 0.03. The proposed algorithms were implemented in MATLAB language. The running time using Intel (R) Pentium D, 3 GHz and 160 MB is approximately 90 s.

The crisp duration of activities based on different value of α are shown in table II. In addition, another computational results based on different values of α , are shown in table III.

Model results show that high level of acceptable risk results in short activities, short inexpensive project and vice versa. In addition, the third mode was selected which has least duration for activities 1 and 7 based on all the levels of acceptable risk. There is not any activity in parallel to activities 1 and 7. So their durations have direct relation with the project time. Selecting the third modes for these activities indicates the project time importance. High indirect project cost units per day or high bonus/penalty for early/delayed project completion or both of them are the reasons of the mentioned issue. Different modes are selected for other activities to which at least one activity is in parallel.

Activity times were fuzzy originally; so, fuzzy project cost is expected instead of crisp project cost. So, nine project costs (53910, 47105,42870,40335,37810, 35810, 32595, 29105, and 19575) were used which obtained based on different levels of risk acceptance, to produce fuzzy project cost. Trapezoidal fuzzy number was used for fuzzy project cost. The smallest and biggest project cost which have been obtained based on α =1 and α =0, respectively, were used as a, d (see figure 1). Other two quantities (b, c) were determined as many as the differences between first level of risk acceptance of nine mentioned project cost and obtained risk by the obtained trapezoidal fuzzy smallest number. The obtained fuzzy project cost is shown in Figure 7. The proposed method was implemented in MATLAB language.



4.2. Sensitivity analysis of the first example

An illustrative sensitivity analysis for the application example is presented in this section. Three independent test cases are examined. The first assumes a 10% increase in time for each execution option of activity 2. The optimum duration and cost based on different level of risk acceptance are presented in table IV.

The optimum duration and cost is now increased. The cost increase is mainly attributed to indirect cost and bonus/delay changes (due to project duration increase) than to direct cost increase. The cost is increased but the use options for each activity are not changed almost.

The next case assumes a 10% times reduction in each option of activity 3 combined with a 10% times increase for each option of activity 2. The optimum duration and cost based on different level of risk acceptance are presented in table V.

Project time and cost is also increased in this case. This is because activity 2 is longer than activity 3. The use options for each activity are not also almost changed in this case.

Finally, a 20% increase in the indirect cost is assumed. The optimum duration and cost based on different level of risk acceptance are presented in table VI.

Project time is not changed almost but project cost is increased. This is attributed to increased indirect project cost per day. The use options for each activity are not also almost changed in this case.

Activity	Mode	α=0	$\alpha = 0.125$	α=0.25	α=375	α=0.5	α=625	$\alpha = 0.75$	α=875	α=1
	1	30	26	24	22	20	18	16	14	10
1	2	20	17	16	15	14	13	12	11	8
	3	12	11	10	9	9	8	8	7	5
	1	40	36	34	32	30	28	26	24	20
2	2	28	25	23	22	21	20	19	18	15
	3	20	18	17	17	16	15	14	13	10
3	1	28	25	23	22.	2.1	20	19	18	15

Table 2. The crisp duration of activities based on different values of α

	2	18	16	15	14	14	13	12	12	10
	3	15	13	12	12	11	11	10	10	8
	1	30	27	25	24	23	21	20	19	15
4	2	20	18	18	17	16	15	15	14	12
	3	16	15	14	14	13	13	12	11	10
	1	15	14	13	13	13	12	12	11	10
5	2	12	11	10	10	10	9	9	9	8
	3	10	9	8	8	7	7	6	6	5
	1	25	22	21	20	19	18	17	16	15
6	2	20	19	18	17	17	16	16	15	14
	3	15	14	14	14	14	13	13	13	12
	1	20	18	17	17	16	15	14	13	10
7	2	15	13	12	12	11	10	10	9	7
	3	9	8	8	8	7	7	7	6	5

Table 3. Project time, cost and the proposed optimum execution mode for each activity based on different values of α

A	Time	Cost	U	Itilizati	on opti	ons for	each a	ctivity	7
			1	2	3	4	5	6	7
0	64	53910	3	2	1	1	1	3	3
0.125	58	47105	3	2	1	1	1	3	3
0.25	54	42870	3	2	1	1	1	3	3
0.375	52	40805	3	2	1	1	1	2	3
0.5	44	37810	3	2	1	1	3	3	3
0.625	49	36280	3	2	1	1	1	3	3
0.75	40	32595	3	2	1	1	3	2	3
0.875	37	29105	3	2	1	1	3	2	3
1	25	19575	3	3	1	1	3	2	3

Table 4. Project time, cost and the proposed optimum execution mode for each activity based on different values of $\boldsymbol{\alpha}$

A	Time	Cost		Utiliza	tion op	tions fo	or eacl	h activi	ty
			1	2	3	4	5	6	7
0	67	56175	3	2	1	1	1	2	3
0.125	60	48715	3	2	1	1	1	3	3
0.25	57	46425	3	2	1	1	1	3	3
0.375	55	43220	3	2	1	1	1	3	3
0.5	47	40225	3	2	7	1	3	2	3
0.625	52	38695	3	2	1	1	1	2	3
0.75	42	34205	3	2	1	1	3	3	3
0.875	33	30365	3	3	1	1	3	2	2
1	26	20700	3	3	1	1	3	2	3

Table 5. Project time, cost and the proposed optimum execution mode for each activity based on different $\boldsymbol{\alpha}$

A	Time	Cost		Utiliza	tion op	tions fo	or eacl	h activi	ty
			1	2	3	4	5	6	7
0	67	56175	3	2	1	1	1	2	3
0.125	60	48715	3	2	1	1	1	3	3
0.25	57	46425	3	2	1	1	1	3	3
0.375	55	43220	3	2	1	1	1	3	3
0.5	47	40225	3	2	1	1	3	2	3
0.625	52	38695	3	2	1	1	1	2	3
0.75	42	34205	3	2	1	1	3	3	3
0.875	33	30365	3	3	1	1	3	2	2
1	26	20700	3	3	1	1	3	2	3

Table 6. Project time, cost and the proposed optimum execution mode for each activity based on different level of $\boldsymbol{\alpha}$

A	Time	Cost	Utilization modes for each activity								
			1	2	3	4	5	6	7		
0	64	55190	3	2	1	1	1	2	3		
0.125	51	48250	3	3	1	1	1	2	3		
0.25	54	43950	3	2	1	1	1	2	3		

0.375	52	41375	3	2	1	1	1	2	3
0.5	44	38690	3	2	1	1	3	3	3
0.625	49	36790	3	2	1	1	1	2	3
0.75	40	33395	3	2	1	1	3	2	3
0.875	37	29845	3	2	1	1	3	2	3
1	25	20075	3	3	1	1	3	2	3

4.3. Evaluation of the proposed model for the first example

Artificial intelligence techniques provide a way to obtain good solutions that require less computational effort than mathematical methods. Therefore, the development of a methodology which solve project management problems by applying artificial intelligence is the main objective of many studies. This matter is intensified by the rapid development of computerbased techniques. However, not all of the proposed approaches are robust models for real-world applications, and some issues such as lack of historical data cause these models cannot be implemented in real-life situations. Some models have requirements which are usually hard to fulfil in practical situations, or the presented results are not reasonably optimum. As a result, it is expected that the presented model will be the most competitive algorithms, particularly when the complexity increases. This means that it is necessary to examine other advanced optimization methods and outline how the model performs in relation to state-of-the-art techniques. Thus, the performance of the proposed approach on the case study against those of three widely used approaches (unmodified genetic algorithm, particle swarm optimization and ant colony optimization) was investigated and the results comparison was carried out. These three methods which are heuristic methods known as approximate solutions provided the optimization problem with less ability in quickness and quality. They respectively delivered project cost magnitude of 45710, 45396 and 50435 in longer running period. The average project cost resulted from nine number mentioned in section 4.2 is 37679. It means that the proposed method outperformed three mentioned methods by 18%, 17% and 25% respectively. These percentages prove the potentialities and the benefits offered by the proposed approach which further strengthen the validity of the proposed approach. It is safe to say that the proposed method can be applied to optimization problems in many different project management problems, especially when dealing with the large-size project networks. Because it provides a useful way to handle some problems in a more flexible and more intelligent manner. This claim is supported by theoretical bases for the model and depth of the above mentioned analyses.

4.4. Setting up more examples and the relevant analysis

To carry out the computational and quality analysis of the presented algorithm, a selected list of difficult instances from the standard benchmark instances of the Project Scheduling Problem Library (PSPLIB) were extracted. The extraction was done as follows. Only the problems with large number of activities were considered for reporting. Then, we employed the method to solve these instances. It was done to gain a clearer understanding of how it performs under varying project sizes and complexities. This experiment demonstrated the model's application successful implementations, because the model offered outcomes fulfilling some expected values for some criteria. This is due to special search capability and structure of the applied method which can offer solutions for problems with large number of activities in an acceptable computational time. It represents that the method performs optimally with increasing data, and therefore, the practical value of the proposed approach is compelling. In addition, for the sake of more evaluation of the model, some real-world examples involving more uncertainties and network complexity were scheduled by using the proposed method. It proved the powerfulness of the model to successfully deal with comprehensive range of different types of projects.

A detailed analysis of how the proposed model perform on the tested large-sized projects was done to expand the sensitivity analysis to include larger, more complex projects. Only in the case of one project whose activities' number of modes was very large, the model had an excellent strategy in practical applications. This exception was due to the fact that the client had not set bonus—penalty structure effectively and economically. It was not both effective to the contractor and feasible to the client. In other words, it was not in nature a coordination mechanism. This caused the fact that the objective function to be extremely dependent on the status of activities' modes, namely the needed expense and time. Nevertheless, this solving and analyzing these mentioned large projects gave the insight that the model can confidently offer quantitative supports for the decision on choosing modes of activities. Because, results from the sensitivity evaluation indicate that the method can be reliably applied to actual projects of all size and complexity in terms of accuracy and solution efficiency. This was due to high ability of the model to work around difficulties associated with previous models. All in all, this observed performance of the algorithm on projects of all size and complexity illustrated the scalability and robustness of the model.

5. Conclusion

The time–cost trade-off problem, which is encountered in project planning, has been extensively studied since the 1960s and several solution methods have been proposed but no one has been implemented in practical applications. Some project characteristics, such as activity duration uncertainty and bonuses/penalties for early/delayed project completion, have not been widely studied in earlier works. As a result, the application field of these methods is narrow and they are of little use for real projects. The present work aims to join such issues in the analysis and to develop a method for making ideal project time–cost decisions applicable to real projects. A simple modification changed the focus of the basic model from minimizing the total cost subject to a project deadline, to a model minimizing project cost. Trapezoidal fuzzy numbers were used to represent the activity durations in a project network in uncertain environment. An index was used to transform fuzzy activity duration to crisp duration. Using afore-stated index and project manager's level of risk acceptance, fuzzy activity duration

turns to crisp duration.

The variables of the program are zero-one variables for the alternative time—resource combinations of each activity. The objective function includes the execution costs of the project activities, the project general expenses (indirect costs), and any applicable bonus or penalty factor. Using the proposed model, information on the status of the project network is ready to avail and sensitivity analysis can be conducted by varying the durations of activities and other parameters, and, the main advantage of the model is considering the project manager's level of risk acceptance. Based on different level of risk acceptance, different activities duration and in result, different schedules could be expected. The project manager select his preferred options based on his own level of risk acceptance. It was concluded that the developed fuzzy scheduling method can help project managers in evaluating different options and minimizing project cost and the risk of the project of being late and overestimated, in the uncertain project execution environment. All in all, it was developed in order to overcome the existing gaps in the literature. These issues assure its straightforward applicability to real construction projects.

The studied model in this work is only a snapshot of planning and scheduling at a fixed time allowing us to examine the framework and the methodology associated with the practical application on construction sites. Clearly, this framework can be applied to a number of instances where for example the operations in the workplace may be increased, decreased or replaced by a set of new operations because of project scope change. Not to mention that these changes may require other activities (e.g. risk analysis) with other different resulted activities. This quantitative and qualitative evaluation of activities in different circumstances that project manager encounters during construction phases provides an in-depth analysis of project status. It results in a macro planning analysis such as selecting major strategies, reviewing the design for constructibility improvement, site planning for major operations and construction paths. This planning is considered to be one of the major operations that can greatly influence the overall success of the project.

The method and MATLAB language were applied to a number of test cases with varying project structure and size (number of activities) and activity time—resource options (the computational efficiency of the model depends mainly on these factors). Results from the evaluation show that the method can be reliably applied to actual construction projects in terms of accuracy and solution efficiency, and after using it, other problems in the construction management field such as safety management and productivity can be confidently solved. These problems themselves determine projects' details such preparation of structural works, e.g. unloading, storing and preparing construction materials. More importantly, practical and major implications and novelties of this study which are because of different reasons such as improved version of an evolutionary algorithm, causes one to be able to use its findings as an efficient alternative tool for a large and important class of problems in project management. Despite this fact, it has some drawbacks which make it necessary to do additional research. For example, this work did not study other project characteristics, such as external time constraints for particular activities. As a result, the application field of these methods is narrow and the methods are of little use for real life projects. Thus, considering important and widely discussed issues in scheduling problems can be added to this method to determine robust and proactive solution.

Future research should focus on an efficient method for optimization project scheduling, considering resources-related issues such as day to day fluctuation in resource demands to cope with constraints of a practical construction project. There are various variations of this kind of problem (resource constrained project scheduling problem) with different objective functions, resource requirement and resource availability patterns. The purposes in these problems are diverse, like full exploitation of non-renewable resources and efficient allocation of scarce resources, which are usually accompanied by major objectives like minimization of project's makespan. In other words, the mode improvement procedure in genetic algorithm is in both directions. However, these aims are not limited to optimization like the mentioned constant availability of resources over time which ensures that the availability of non-renewable resources is used completely, but fulfillment of some limitations (renewable resources are usually available in limited quantity and sum of resources needed on any day by all activities scheduled on that day should not exceed availability of any resource type) and avoiding some complications like incurring idle resources costs (caused by contractors because of scheduled interruptions of the selected crew) could be contributions of studies. They could be valuable additions and validate the effectiveness of the proposed algorithm.

As mentioned before, reduction of project's completion time is of crucial importance. On the other hand, resource constraints limit the resource demand imposed by the activities being processed at a time to the available capacity. That is to say, it is necessary to generate a precedence and resource feasible active schedule. But, not all precedence relations between activities deterministic. The variant is to split some activities in order to minimize the time required to complete all the mentioned activities which are repetitive. Obviously, problem in which each activity can be split at discrete time instants under the constraints of a maximum number of splitting and a minimum period of continuous execution can be scheduled with better results. Because activity splitting improves robustness, ad with the increase in the maximum number of splitting, the decrease in the minimum execution time, the decline in the setup times, and the extension of the project deadline, robustness rises. In fact, a prerequisite to successfully manage resource availability constraints is to permit the activities to have splitting. However, there are other techniques to acquire promising results in the domain of required processing time of activities, such as productivity maximization. But, when combination of these tactics are applied, it is guaranteed that occupation of non-renewable resources by activities ensures the very high quality of the acquired schedule.

As well as the narrowness of the proposed model, features related to potential computational challenges such as low efficiency of the model in certain occasions should be taken into account. It is difficult to solve for optimality by applying the model when the number of activities in the project increases, or the project is of special network. In the presented method, the mode improvement operation is applied over the generation of new schedules on some of the activities in order to improve the trend of solution quality. This action is a troublesome duty or without any acceptable outcome. This is to say, the algorithm does not work well on all of problems. This is because of the fact that interrelation between parameters in some

projects disrupts obtaining convergence during the model run. In order to removes this fault, some strategies such as modification of the the search space during running the model are advisable. In addition, the issue of selecting an appropriate representation for different matters is crucial for the search. It is worth mentioning that appropriate representation scheme can lead to improvements such as reduction of complexity and required expertise for programming. Moreover, in order to measures the quality of solution (generated schedules), a simple efficient fitness function into which many issues can be moved should be defined. All these mentioned solutions were discovered after carrying out some computational analyses.

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