

# **The Appropriate Statistical Distribution for Residuals in Seismic Attenuation Relationships**

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## **Abstract**

The developed ground motion models are mainly based on the assumption of normality of the residuals. The extreme value distribution is a statistical distribution used in modeling rare events and extreme scenarios. In large earthquakes with a long return period, the recorded peak ground accelerations (PGAs) are large and rare, so the assumption of extreme distributions is not unexpected for these accelerations. The extreme value distribution has two conventional forms; generalized extreme value (GEV) for maximum values of blocks with the same time duration and generalized Pareto distribution (GPD) for values above a determined threshold. Due to the less recorded numbers of PGAs, using the GPD distribution in examining the extreme values of the PGAs, is more appropriate. If the GPD distribution assumption for PGA data be accepted, it is suggested to develop a seismic acceleration attenuation relationship for large or extreme data based on the GPD distribution, and the common assumption of lognormal distribution is discarded. This article reviews the statistical distributions used in ground motion models. The results suggests that in the development of ground motion relationships, the normal distribution for residual be abandoned with a fundamental revision and the next generation of these models should be developed based on the GPD distribution.

**Keywords:** Ground motion attenuation models, Extreme value theory, Generalized extreme value distribution, Generalized Pareto distribution, Residuals.

## 1. Introduction

In most ground motion models, the assumption of normal distribution for residuals and consequently logarithmic transformation for intensities are used. In many studies, this assumption has not been checked, and in some studies, the criterion for checking the assumption of normality of residuals has not been chosen correctly.

As the first model in the technical literature review of ground motion models, we can refer to the developed model by Milne and Davenport (1969) which used data collected by Esteva and Rosenblueth (1964) [1, 2]. This model with an exponential form and in terms of gravity acceleration (g) is developed as following:

$$A = \frac{a_1 e^{a_2 M}}{a_3 e^{a_4 M} + \Delta^2} \quad (1)$$

Where, A is in percentage of g,  $a_1 = 0.69, a_2 = 1.64, a_3 = 1.1, a_4 = 1.1$ , M is the earthquake magnitude and  $\Delta$  is the epicentral distance of the event.

Blume (1977), Donovan and Bornstein (1978), Esteva (1970), Faccioli (1978), Milne (1977), and Orphal and Lahoud (1974) attempted to develop ground motion models using exponential form [3-8].

Although McGuire (1977) used the exponential form for the ground acceleration model [9]. The logarithm form for the ground motion acceleration model was firstly used by Denham and Small (1971) which is still used today [10]. Also, Ambraseys (1975) and McGuire (1978) used the logarithm function for two sides of the previous ground motion relationship [11, 12].

## 2. The Historical Development- The Normal distribution model for residuals

The first report about investigating the normal distribution for residual values against distance, square of the seismic magnitude, and square of predicted acceleration in ground motion models is presented by Campbell, (1981) [13]. The normal distribution for residuals, and the Kolmogorov-Smirnov test with a 90% confidence support this claim that PGA values have a lognormal distribution. Also in this model, the residual values against distance and magnitude intervals are presented, and no difference in its form was observed in different magnitude intervals. In addition, the effect of different values of hypocentral depth on residuals and the correlation of coefficients of the ground motion model to hypocentral depth are investigated. McCann Jr and Echezwia (1984) after developing four ground motion models tested the

assumption that the logarithm of the residuals is normally distributed, which was not rejected by the observations [14]. Kawashima et al. (1986) were among the first to draw the histogram of residuals, and fit a normal distribution to it [15].

In a report by Campbell (1990) weighted normalized residuals against hypocentral depth, distance, and magnitude are investigated [16]. Crouse (1991) plotted residuals against magnitude and find uniform distribution [17]. Also, Crouse and McGuire (1996) checked plots of residuals and found a uniform distribution for them [18].

Theodulidis and Papazachos, (1994) with investigating the histogram of residuals fitted normal distribution on them [19]. It is mentioned by Abrahamson et al. (2002) and Bommer and Abrahamson (2006) that the lognormal distribution in ground motion models makes overestimation of ground motion intensities [20, 21]. Therefore, creating a limit and truncation lognormal distribution was proposed to avoid overestimation, but selecting the truncation point was difficult [22]. In a report by Ólafsson et al. (2001) about stochastic ground motion models in Iceland, the residuals of acceleration were calculated. The basic assumption was the normal distribution for residuals in developing models and with the Kolmogorov-Smirnov and Chi-square tests, the assumption of the normal distribution of the residuals was confirmed with a 5% significance level [23].

Douglas and Smit, (2001) examined the normality of residuals on different data sets and conducted the lognormal distribution is appropriate for all periods and data [24]. Bommer et al., (2004) pointed out that the residuals can be considered lognormally distributed except the tail part of the distribution [25]. Yamada et al., (2009) fitted the lognormal distributions for PGAs and the uniform distribution for PGDs [26].

Tavakoli and Pezeshk, (2007) supposed the normal distribution with zero average for residuals. In this paper, the histogram of residuals against magnitude and hypocentral depth was presented and no specific trend was observed in the residuals [27].

Graizer and Kalkan, (2007) presented a ground motion model for horizontal PGAs for shallow earthquakes [28]. The plots of residuals against magnitude, distance, and Vs30 were presented. This paper does not mention the histogram of residuals and its analysis. In a report by Baker, (2008) about an introduction to probabilistic seismic hazard analysis (PSHA), the normal distribution with zero average is supposed for residuals [29]. Rezaeian and Der Kiureghian,

(2010) in their report noted that the observed values of the residuals are normal with a zero mean [30].

Graizer, (2010) using the Atlas database of shallow crustal events extended and tested previous Graizer-Kalkan ground motion attenuation models [28, 31, 32]. In this paper, the normal distribution with zero mean is considered.

Most studies in the field of developing ground motion models assume a normal distribution of residuals [25, 33, 34].

Soghrat et al. (2012) investigated strong ground motion in northern Iran using a specific barrier model. In a part of this research, the histograms of residuals for two periods, 0.1 and 1 sec, are plotted and the normal distribution is fitted on them [35].

Bradley and Hughes, (2013) investigated a spatially distributed ground motion intensity map and its application in liquefaction. In this paper, the normal distribution for residuals with zero mean is supposed [36]. Azarbakht et al. (2014) presented a new method for stability of ground motion models. In this paper, for intra and inter-event residuals, the normal distribution is supposed [37]. Akkar and Bommer, (2010), Atkinson and Boore, (2011), Cauzzi and Faccioli (2008), Harris and Abrahamson (2014), Jaimes et al. (2016), and Kowsari et al. (2016) used zero mean normal distribution for residuals [38-43].

Boore et al., (2014) presented ground motion models in the NGA-West 2 project. In this paper to validate the equation only the plot of residuals against variables has been used and any assumption about residual distribution and the histogram of residuals has not been discussed [44].

Mousavi et al., (2014) using the P-value analysis showed there is no trend for residuals against distance and  $V_{s30}$ . In this paper for residuals, the normal distribution with zero mean is selected [45].

Tusa and Langer, (2016) investigated variables of ground motion models for the volcanic area of the Mount Etna. However, despite the mentioned suppose that the histogram of residuals has the normal distribution form, a skewness is observed in this histogram. The authors in this paper, to validate the observations used the Lilliefors normal test and its results show the normal distribution for PGAs residuals in shallow data (focal depth less than 5 km) and depth data (focal depth more than 5 km) for the significance level 1 and 5%, respectively [46].

Zanini et al. (2016) supposed normal distribution with mean and standard deviation equal to 0 and 1, respectively [47].

Bindi, (2017) used residual analysis to predict the power of ground motion models. However, this study did not mention the distribution of residuals and their dispersion against the main variables such as the magnitude and distance [48].

In developing the ground motion prediction equation for northern Iran data by Soghrat and Ziyaeifar (2017) just the normal distribution of residuals is pointed but no research has been conducted on the histogram of the residuals and its relevant assumptions [49].

### **3. Statistical Analysis & Recommendations- The appropriate distribution of ground motion data**

In seismic hazard assessment, the development of ground motion models to predict PGA, PGV, and PSA is important. The development of these models is based on their input data. In this data, the big ground motion intensities, which are more important, describe the tail part of the fitted distribution on seismic data.

Since the occurrence probability of large ground motion intensities is low, a set of statistics distributions based on the extreme value theory is applicable [50]. Moreover, statistics distributions of extreme value theory can be used replace of the lognormal distribution in developing ground motion models. Replacing the distribution makes a change in the functional form of the ground motion model.

In some studies, some details are provided about how the distribution of residuals does not follow the normal distribution.

Ambraseys et al. (2005), and Cauzzi and Faccioli, (2008) investigated the appropriateness of the logarithmic transformation of ground motion intensities. Their results showed that despite the weakness of this logarithmic transformation in some periods, this transformation is considered acceptable because it is accepted in some adjacent periods [40, 51].

Huyse et al. (2010) showed that the tail of PGA data and the residuals distribution of them is not lognormal. Their findings suggest the use of a GPD distribution for the tail part of PGAs [52].

McBean et al., (2015) showed statistical distribution functions such as PGA and PGV in a strike-slip fault, are not lognormal. They used the Kolmogorov-Smirnov test and their results proved that the lognormal distribution for PGAs is inappropriate [53].

Abrahamson et al. (2016) applied the normal assumption of residuals. They showed the tail part of the distribution of inter-event residuals is not a normal distribution but this issue is not necessary in engineering levels [54].

Dupuis and Flemming, (2006) modeled PGAs and PSAs in different periods using GEV distribution and demonstrated that maximum values of an accelerograph should fitted by an appropriate distribution than lognormal [55].

Raschke, (2013) discussed many seismology and statistical assumptions and developed a ground motion model for PGAs. He identified extreme value theory for the definition of ground intensity relationships in the probabilistic seismic hazard assessment and criticized the normality of residuals. Moreover, this paper proved a wrong choice of ground motion intensities could be effective in the probabilistic seismic hazard assessment [56].

Pavlenko (2015, 2017) investigated the effect of alternative distribution in the seismic hazard assessment and criticized the common assumption of lognormal distribution for PGAs. Their results showed the appropriateness of the GEV distribution for the central part and GPD distribution for the tail part of the observations [57, 58].

In previous studies, although the GEV distribution is used but its primary condition, the block maxima, has not been applied properly. While, for selecting the extreme values in GEV distribution using the block maxima method is necessary. In fact, in the mentioned studies, the GEV distribution is only used as a skewness distribution without considering the initial steps of the extreme value theory. They used the GEV distribution as a selected distribution to model positively skewed data such as lognormal, gamma, inverse Gaussian, etc.

In Borzoo et al., (2020a) the GEV distribution is used as a statistical tool to model ground motion intensity data, and its features and applications are described [59]. In this paper, the GEV distribution with the lognormal distribution, as the common distribution in seismic acceleration, and the normal distribution as a basic distribution are compared. The comparison results using three AIC, BIC, and Log-likelihood criteria showed the GEV distribution is the most appropriate distribution for PGA and PSAs in lower periods, and on the other hand, PGV and PSAs data in upper periods follow lognormal distribution. These results in two conditions of near-source and far-source data are the same (Tables 1 and 2).

Table 1 Distribution fit for near-source data [59]

-log-likelihood	AIC	BIC
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	GEV	lognormal	normal	GEV	lognormal	normal	GEV	lognormal	normal
PGA	<b>-1584.140</b>	-1509.210	-599.614	<b>-3164.280</b>	-3016.420	-1197.228	<b>-3153.389</b>	-3010.975	-1191.783
PSA (T = 0.2 s)	<b>1102.790</b>	1108.170	2021.560	<b>2209.580</b>	2218.340	4045.120	<b>2220.471</b>	2223.785	4050.565
PSA (T = 0.3 s)	<b>1021.420</b>	1036.460	1859.970	<b>2046.840</b>	2074.920	3721.940	<b>2057.731</b>	2080.365	3727.385
PSA (T = 1 s)	-240.520	<b>-278.954</b>	579.665	-477.040	<b>-555.908</b>	1161.330	-466.149	<b>-550.463</b>	1166.775
PSA (T = 1.2 s)	-597.282	<b>-641.797</b>	277.894	-1190.564	<b>-1281.594</b>	557.788	-1179.673	<b>-1276.149</b>	563.233
PGV	6648.340	<b>6611.670</b>	7609.090	13300.680	<b>13225.340</b>	15220.180	13311.571	<b>13230.785</b>	15225.625

Table 2 Distribution fit for far-source data [59]

	-log-likelihood			AIC			BIC		
	GEV	lognormal	normal	GEV	lognormal	normal	GEV	lognormal	normal
PGA	<b>-834.872</b>	-791.520	-717.079	<b>-1665.744</b>	-1581.040	-1432.158	<b>-1658.092</b>	-1577.214	-1428.332
PSA (T = 0.2 s)	<b>-147.253</b>	-136.992	-6.278	<b>-290.506</b>	-271.984	-10.555	<b>-282.854</b>	-268.158	-6.729
PSA (T = 0.3 s)	<b>-125.560</b>	-124.136	-43.705	<b>-247.120</b>	-246.272	-85.409	<b>-239.468</b>	-242.446	-81.583
PSA (T = 1 s)	-216.901	<b>-232.272</b>	-128.093	-429.802	<b>-462.544</b>	-254.186	-422.150	<b>-458.718</b>	-250.360
PSA (T = 1.2 s)	-278.182	<b>-294.482</b>	-155.658	-552.364	<b>-586.964</b>	-309.316	-544.712	<b>-583.138</b>	-305.490
PGV	1060.990	<b>1052.620</b>	1135.170	2125.980	<b>2107.240</b>	2272.340	2133.632	<b>2111.066</b>	2276.166

In addition, three developed ground motion models using the three mentioned distributions are compared to find the appropriate distribution for PGAs. The accuracy of the models was investigated using residual analysis. The results shown in all the near-source and far-source earthquakes, the root mean square error (RMSE) of residuals using the GEV distribution is smaller than the lognormal distribution (Table 3).

Table 3 The RMSE and standard deviation of residuals [59]

Ground-motion data		RMSE of residuals			Standard deviation of residuals		
		GEV	lognormal	normal	GEV	lognormal	normal
Near-source earthquakes	1999 Chi-Chi	<b>0.1232</b>	0.1406	--	<b>0.1235</b>	0.1401	--
	2008 Iwate	<b>0.1977</b>	0.1998	--	0.1984	<b>0.1947</b>	--
	1994 Northridge	<b>0.1350</b>	0.2010	--	<b>0.1355</b>	0.2012	--
	1989 Loma Prieta	<b>0.1281</b>	0.1780	--	<b>0.1269</b>	0.1734	--
	1987 Whittier Narrows-01	<b>0.0739</b>	0.1218	--	<b>0.0743</b>	0.1203	--
Far-source earthquakes	2007 Chuetsu-Oki	<b>0.0134</b>	0.0136	--	0.0135	<b>0.0131</b>	--
	2004 Niigata	0.0163	<b>0.0136</b>	--	0.0163	<b>0.0132</b>	--
	2000 Tottori	0.0224	<b>0.0155</b>	--	0.0224	<b>0.0153</b>	--
	2008 Iwate	<b>0.0276</b>	0.0282	--	<b>0.0269</b>	0.0282	--

1999 Chi-Chi	0.0267	0.0299	<b>0.0243</b>	0.0246	0.0294	<b>0.0245</b>
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The GEV distribution reduces the average value of RMSE and the standard deviation by about 19% and 18%, respectively. It is mentioned by the authors that using the GEV instead of the lognormal distribution improves the ground motion models, especially in large intensities.

In Borzoo et al., (2020b) the GPD distribution for different ground motion intensities is investigated (Table 4) [60]. In different thresholds and for selected ground motion parameters, the GPD is demonstrated more appropriate than the lognormal distribution.

Table 4 The proper distribution for various thresholds on PGA data [60].

Threshold (g)	-Log-Likelihood		AIC		BIC		Observations above threshold (%)	Number of observations above threshold
	Log. D.	GPD	Log. D.	GPD	Log. D.	GPD		
0.05	-2035.41	-2347.82	-4068.82	-4691.64	-4063.19	-4680.39	13.24	2051
0.10	-923.53	-1145.67	-1845.06	-2287.34	-1839.97	-2277.17	7.69	1192
0.20	-271.16	-387.90	-540.31	-771.80	-536.06	-763.30	3.35	519
0.30	-104.86	-163.12	-207.72	-322.24	-204.13	-315.06	1.73	268
0.40	-46.44	-87.68	-90.88	-171.35	-87.83	-165.25	1.01	156
0.50	-20.34	-39.58	-38.69	-75.15	-36.20	-70.17	0.57	89
0.60	-12.42	-30.81	-22.83	-57.62	-20.72	-53.40	0.39	61
0.70	-4.27	-20.88	-6.54	-37.75	-4.87	-34.42	0.25	39
0.80	0.21	-4.87	2.42	-5.74	3.41	-3.75	0.13	20

As expected, the superiority of the GPD with increasing the threshold is more obvious. Accordingly, to generate intensity maps for extreme scenarios, a ground motion model that is developed using the GPD distribution should be considered. Also, Borzoo et al., (2019) demonstrated that the appropriate distribution for seismic magnitudes is GPD distribution [61]. For more details about the Practical Implementation of EVT, Block Maxima, and Peak Over Threshold methods using actual seismic datasets refer to [59, 60].

#### 4. Conclusion

The developed ground motion models are mainly based on the assumption of normality of the residuals. One of the notable results in this paper is the superiority of the GEV distribution compared to the lognormal, especially at high intensities. This issue can lead to the formation of a new generation of ground motion models based on extreme value distributions or other



statistical distributions. Hence, a suggested model is the use of the GEV distribution for the middle parts of the data and the use of the GPD distribution for the tail parts of the data. This two-component mixed distribution, based on the mentioned results, is superior to the common lognormal distribution used in existing ground motion models. Also, if a study is focused on critical and extreme ground motion intensities, we should explicitly use the GPD distribution to model these intensities. Therefore, it is suggested that in the development of ground motion relationships, the normal distribution for residual be abandoned with a fundamental revision and the next generation of these models should be developed based on the GPD distribution.

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